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ELECTRICAL AND PHOTO-ELECTRIC PROPERTIES OF THIN FILMS OF RUBIDIUM ON GLASS

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ABSTRACT

Films which spontaneously deposit on glass surfaces in a highly exhausted cell containing rubidium are electrically conducting, and photo-electrically active. A study of the photo-electric properties of a rubidium coated plane glass surface shows the normal and selective effects less well differentiated than for the similar coatings which form on metal plates. A rubidium film formed on the inside of a glass cylinder is found to exhibit, in the dark, a pure ohmic resistance. This decreases under illumination in a manner which appears to be explained as due to the liberation of photo-electrons which under a potential gradient form an added current along the tube.

INTRODUCTION

In a previous paper a description was given of the photo-electric properties of the thin films which deposit spontaneously on clean metal surfaces in highly exhausted inclosures containing alkali metal.[1] The present work is virtually a continuation of that previously reported, but deals with thin films of one of the alkali metals, rubidium, spontaneously deposited on a glass surface.

This work had its origin in the observation that high vacuum photo-electric cells, in which the alkali metal is rubidium, always give trouble from large leakage currents. To prevent these leakage currents, it has been found necessary to wrap heating coils around appropriate portions of the cells in order to drive the

[1] Ives, *Astrophysical Journal*, 50, 209, 1924.

rubidium coating from the surface. It was observed that the glass walls of some of these rubidium cells, being covered with conducting, although usually quite invisible rubidium films, were photo-electrically sensitive. This observation prompted the construction of a special photo-electric cell, similar to the cells used in the previously reported investigation, with the one exception that the plate on which light was allowed to fall and from which the photo-electric current was observed, was made of glass instead of metal. The photo-electric properties of the rubidium layer on this glass plate were then studied in the same way as the photo-electric properties of alkali metal films on polished metal plates were studied previously. Some additional observations on the electrical resistance of thin rubidium films on glass, both in the dark and when illuminated, were included in the study and are here reported.

APPARATUS

The special photo-electric cell constructed for this study is shown diagrammatically in Figure 1. The cathode consists of a plate of black glass, which is optically ground and polished on one face. Platinum wires are sealed into the two ends of this plate and are connected to wires leading out at each end of the tube. A tungsten filament placed close behind the glass plate makes it possible to warm the plate by radiant heat. The anode consists of a nickel cylinder with a narrow rectangular opening in front. In order to secure illumination at normal incidence with a minimum of back reflection, a glass tube with a large diameter bulb forms the window for normal illumination. The back of the nickel cylinder anode is also cut away in order to reduce the amount of anode surface illuminated by stray light. Heater coils on the two ends of the tube are provided to secure insulation, for the reason already given. Illumination of the plate was by "white" light, from a helical coil tungsten lamp. Measurements were made in general according to the methods described in the previous paper above referred to.

ELECTRICAL PROPERTIES

When the glass plate is warmed by long exposure to the radiant heat of the tungsten filament behind, it becomes entirely free from rubidium coating, as is shown by the fact that its resistance is prac-

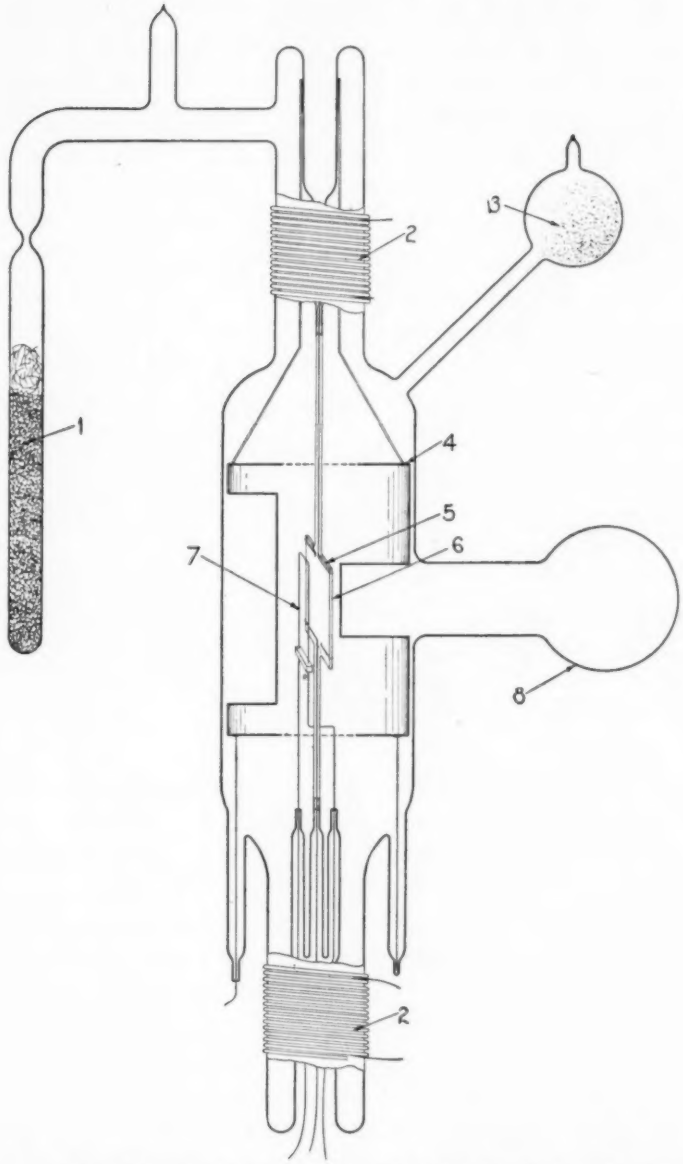


FIG. 1.—Photo-electric cell in which the cathode is a black glass plate, on which rubidium spontaneously deposits. (1) charcoal tube; (2) heater coil to prevent surface leak; (3) filling tube; (4) metallic can: anode; (5) wire fused into glass plate: one at each end of plate; (6) black glass plate; (7) filament; (8) bulb directly in front of plate to prevent multiple reflections at normal incidence.

tically infinite, and it is photo-electrically insensitive. After the plate has cooled, although for some hours its appearance is entirely unchanged, and no trace of deposited metal can be detected by examination with a microscope, the path composed of the two lead-in wires and the glass plate becomes conducting. When the current is measured for various voltages, a straight line is obtained, indicating that the resistance is a simple ohmic one. The resistance of the plate used, of which the dimensions were 19×40 mm, is 1.3×10^8 ohms. This resistance is so large that rubidium leaks of this character might

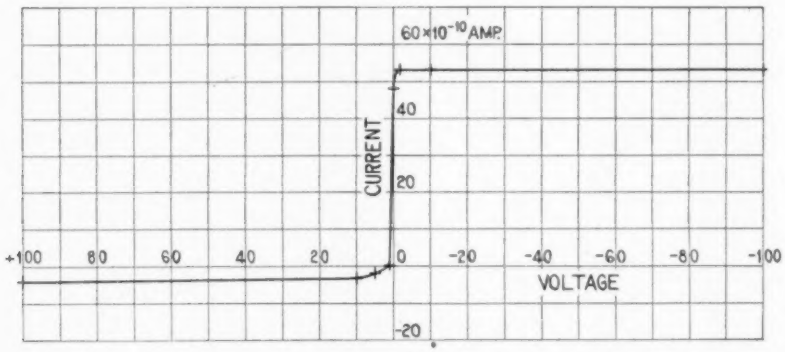


FIG. 2.—Voltage-current relation exhibited by photo-electric current from glass plate.

be useful as high resistances in connection with electrometers and other measuring devices.

PHOTO-ELECTRIC PROPERTIES

Figure 2 shows the curve obtained for voltage and photo-electric current when the glass plate with its fully formed rubidium coating is illuminated. It will be seen that this is a saturation curve of the normal type, indistinguishable from those obtained with the alkali metal coatings on polished platinum. In order to find the relative magnitude of this current as compared with that from a rubidium layer on a platinum plate, measurements were made under the same condition of vertical illumination on this cell and on a platinum plate cell of exactly similar design and dimensions which had been used in the previous work. The result of this measurement shows that the glass plate cell gives a current approximately $2\frac{1}{2}$ times that

from the metal plate. This difference might possibly be due to other differences in the cells, but it is consistent with other photo-electric characteristics, as will be shown shortly.

Carrying through the same series of experiments as those performed on the metal plate cells, observations were made of the growth of photo-electric sensitiveness with time, starting with the glass plate cleaned off by heating, and the walls of the cell held at a low temperature by a layer of carbon dioxide snow. These observations were made with the light incident at about 70° from normal, and with plane polarized light; the electric vector being oriented alternately parallel and perpendicular to the plane of incidence, as is conventionally done to bring out the difference between "normal" and "selective" photo-electric effect, which is so striking a feature of films of alkali metal deposited upon metal plates. The results of the series of observations obtained as the glass plate cooled are shown in Figure 3. The topmost curve shows the current between the terminals on the plate when a potential difference of 10 volts was applied. The conductance reaches a steady value, for the conditions of this experiment, in about forty minutes. The curve in the middle of the figure shows the history of the photo-electric current when the plate is illuminated by polarized light, with the electric vector parallel to the plane of incidence. The current rises quickly from zero value, attains a maximum after about five minutes, declines slightly thereafter, then rises to a second maximum at about twenty minutes, declining slightly from there to its steady value. The photo-electric current due to light polarized with the electric vector perpendicular to the plane of incidence is shown in the lowest curve. It shows a small maximum at five minutes, with possibly an indication of a second maximum at twenty minutes.

Considering the photo-electric curves in their relation to the conductivity curve, it appears probable that the double maximum of the "selective" photo-electric current curve is due to the combination of a maximum in the photo-electric effect with a rapid variation in the amount of rubidium on the surface, as evidenced by the conductivity change. The first maximum is probably intrinsically very high indeed, similar in fact to the high maximum obtained in the early stages of the accumulation of alkali metal films on

platinum, as described in the paper already referred to. It is, however, kept down by the small amount of surface covered. The decrease following this maximum (assuming that the general course of the phenomenon is as in the metal plate cells) is at first only partly counteracted by the increase in the amount of alkali metal deposited; then for a period it is much more than offset by this increase, caus-

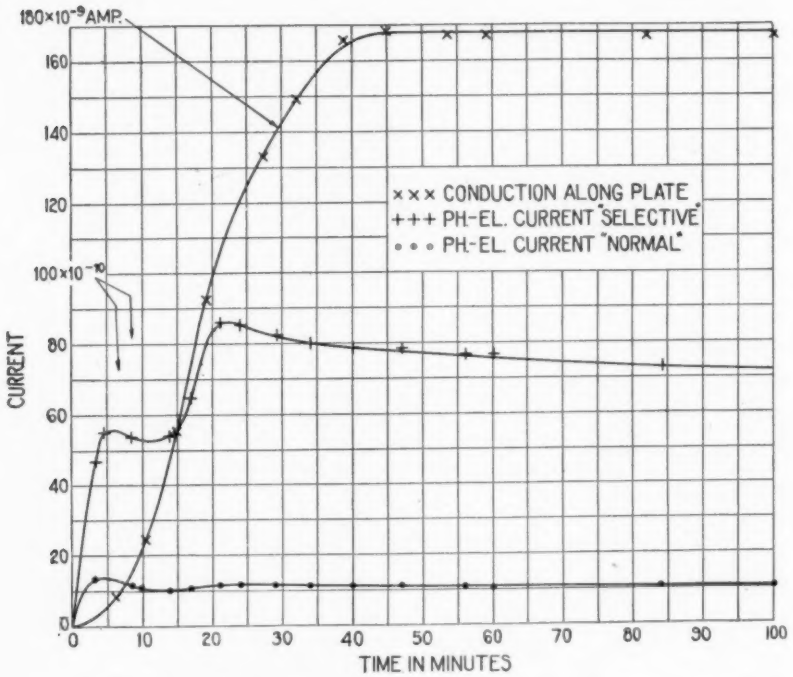


FIG. 3.—Variation of photo-electric current from glass plate, and of current between terminals on plate, with time, as rubidium film forms.

ing a second maximum, after which, the rubidium film having reached a stable condition, the photo-electric effect continues its slow decline to its final value. In the case of the "normal" photo-electric current, these two maxima are only just visible, indicating a different rate of rise and fall from the photo-electric maximum, as was the case with the metal plate cells.

In Figure 3 the greatest ratio of "selective" to "normal" effects is 8 to 1, which is less than half that frequently obtained with the

thin films on metal, but much greater than the ratio found with massive alkali metal. This ratio, 8 to 1, is, however, greater than was usually found in experiments with the glass plate cell, when special attention was paid to the diaphragming of the light beam for both positions of the nicol. Ordinarily this ratio has been found to be about 5 to 1. More complete information on the question of "normal" and "selective" characteristics is furnished by the "angle" curves shown in Figure 4. The curve for illumination with the electric vector parallel to the plane of incidence shows a sharp maximum for illumination incident at 70°; the whole curve being similar in shape to those obtained with the metal plate cells. The curve for light polarized in the other plane is, however, quite different from those characteristic of the thin films on metal plates. Instead of a steady decrease in value as the angle of incidence is increased, the photo-electric current increases slowly up to an angle of 50° and then drops off rapidly.

The behavior of the photo-electric current, as produced by illumination in the two planes of polarization, appears to be capable of explanation on the assumption that the photo-electric effect is, for all angles of illumination and polarization, a mixture of normal and selective. Thus, let us suppose that the metal lies upon the glass surface in the form of a scattered assemblage of small clumps or particles, so that the metal surfaces are not everywhere parallel to the glass surface. When the electric vector is normal to the plane of incidence of the light upon the glass it meets the surfaces of some of these particles at an angle, so that there is present some "selective" effect. It is only necessary to assume that a small part of the metal surface deviates from parallelism with the glass. Under these conditions, while the "selective" angle curves would be little affected because of the relatively large magnitude of this effect, the "normal" curves would be partially warped toward the characteristic selective shape. Coincidently the ratio of "selective" to "normal" would be decreased, chiefly by the enhancement of the "normal" effect. Such a surface could appear, judged from a single measurement of the "selective" to "normal" ratio, to exhibit no selective characteristic. Determination of its "angle" curves would, however, show it to differ in its photo-electric properties from one whose elements did

not individually exhibit the selective effect, by the peculiar character of the "normal" curve.

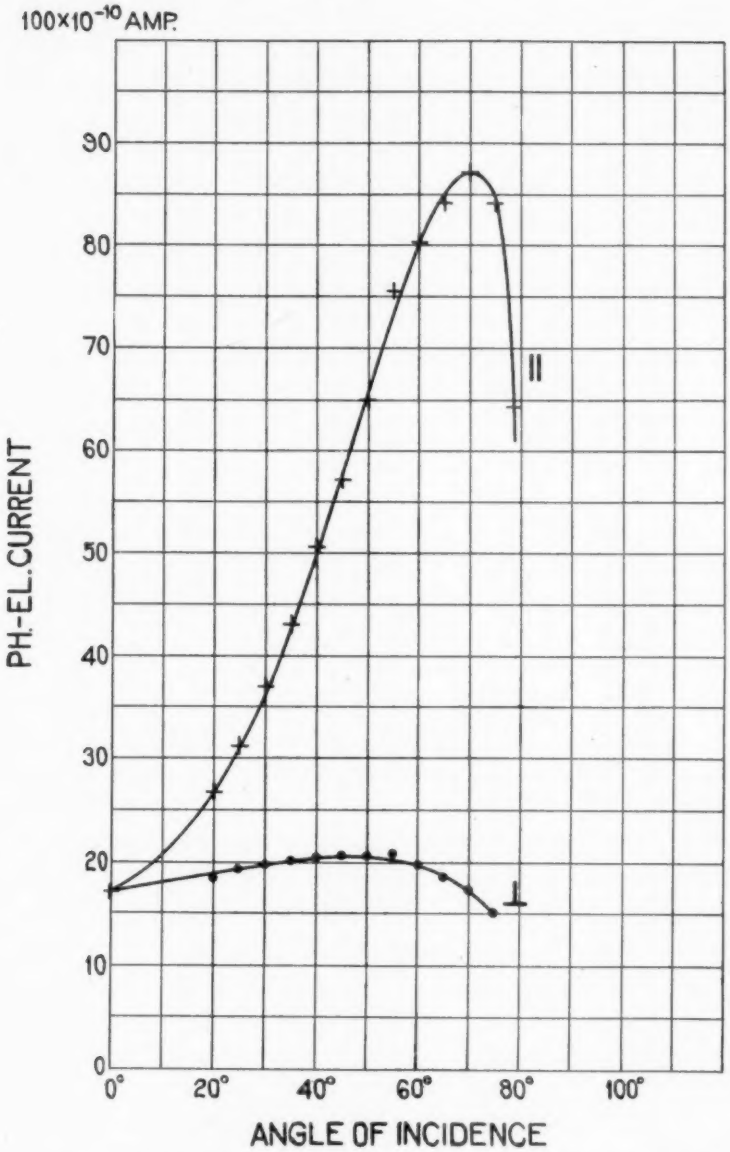


FIG. 4.—Photo-electric currents for various angles of illumination, and two planes of polarization.

Further evidence that the deposit of rubidium on glass consists of clusters of atoms, rather than a uniform layer, is offered by the character of the deposit after the glass surface has stood for days or weeks. After such an interval the glass surface no longer looks perfectly clean, but is definitely speckled with microscopic crystalline aggregates of alkali metal, as are also the glass walls of the cell. It is therefore likely that in the early stages of deposition the rubidium is present in sub-microscopic specks, presenting faces at various angles to the supporting glass plate.

APPARENT PHOTO-RESISTANCE EFFECTS

Rubidium leak cells may be constructed of such a form as to exhibit, under illumination, effects which are a combination of their

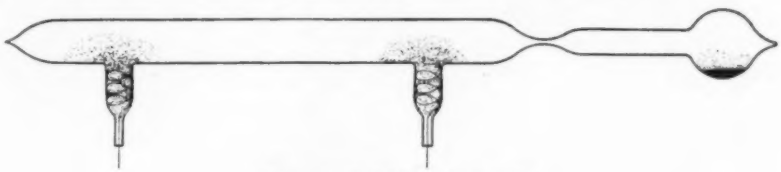


FIG. 5.—Rubidium leak cell

behavior as ohmic resistances and photo-electrically sensitive surfaces. An experimental form of tube in which both ordinary electrical conductivity and photo-electric emission may be observed together is shown in Figure 5. This consists of a glass tube with electrodes sealed in its two ends and containing a few globules of rubidium. The center portion of the tube is flamed to drive off all visible traces of alkali metal.

The current voltage characteristic for this cell in the dark is a straight line (Fig. 6) indicating a pure ohmic resistance. If the central portion of the tube is illuminated, the current increases, recalling the behavior of selenium.

The complete behavior of the added current due to illumination is exhibited in Figure 7, where the added currents produced by "white" light falling through a series of neutral tint screens are shown. The illumination here was very intense, produced by a focused image of a headlight filament. The maximum added current did not under these conditions exceed the "dark" current, as shown by the current values recorded in the figures. It will be seen that for

sufficiently high voltages applied to the ends of the tube, the added current due to illumination is directly proportional to the illumination. For very low voltages, however, this linear relation is not observed, the current apparently approaching saturation. Plotting total current versus voltage, for various values of illumination, we obtain a series of lines each corresponding to one intensity of illumination. These lines, for the higher voltages, are parallel to the straight line shown in Figure 6, as obtained in the dark. At lower voltages, the lines bend down to the origin.

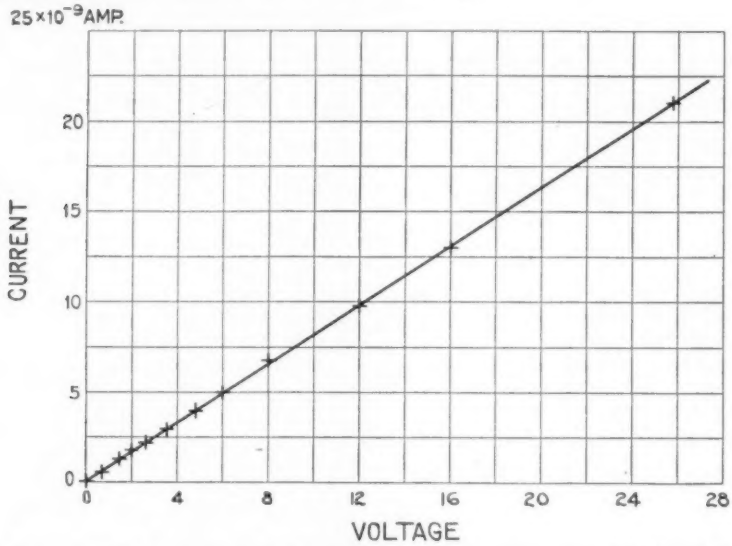


FIG. 6.—Voltage-current relation in rubidium leak cell, measured in the dark

The explanation of the added current due to light appears perfectly straightforward when the higher voltages are considered. It is to be remembered that the illuminated film on the inside of the glass cylinder becomes a source of photo-electrons. If there is no potential gradient along the tube, these photo-electrons strike the other portions of the tube walls equally to either side of the illuminated point and no added current along the tube is observed. When, however, there is a potential gradient introduced, the photo-electrons are directed toward the end of the tube which is positive with respect to the illuminated point, and this figures as an added current, proportional to the illumination. For very small potential gradients, the electrical field becomes distorted by the accumulation of photo-

electrons on the glass surface, thus producing non-linear illumination-current curves of the type found in an earlier investigation,[1] where opportunity was afforded for the charging up of glass walls.

DISCUSSION

The chief interest in these observations of the photo-electric properties of rubidium films on glass lies in a comparison of their behavior with films on a conducting substratum. The outstanding difference is the less pronounced differentiation of the selective and

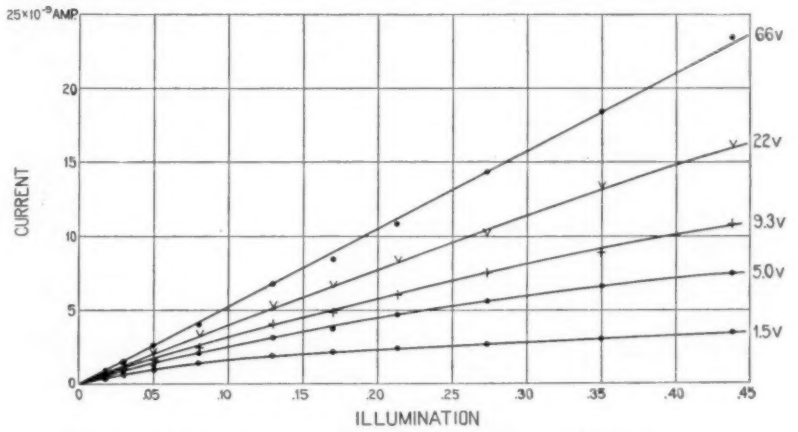


FIG. 7.—Added currents due to illumination of the rubidium leak cell

normal photo-electric effects. In the paper previously referred to, dealing with alkali metal films on metal plates, it was suggested that the selective photo-electric effect is to be ascribed to a definite orientation of the alkali metal atoms. This orientation, according to the evidence given in the previous paper, does not occur in the alkali metal in bulk, but is very pronounced at a certain stage of the formation of the alkali metal film. The phenomena described in the present paper may be interpreted as showing that the arrangement of alkali metal atoms necessary to give the selective photo-electric effect definitely separated from the normal effect can occur only when the surface, to which the atoms attach themselves, is an electrical conductor.

BELL TELEPHONE LABORATORIES, INCORPORATED
August 1925

[1] Ives, Dushman, and Karrer, *Astrophysical Journal*, **43**, 9, 1916.

MEAN DISTRIBUTION OF STARS ACCORDING TO APPARENT MAGNITUDE AND GALACTIC LATITUDE[1]

By FREDERICK H. SEARES, P. J. VAN RHIJN, MARY C. JOYNER, and MYRTLE L. RICHMOND

ABSTRACT

Mean distribution of stars according to magnitude and galactic latitude.—The distribution has been determined from (*a*) *Mount Wilson Catalogue of Selected Areas*, covering $m = 13.5$ to 18.5; (*b*) 33 zones of the *Astrographic Catalogue*, $m = 9.0$ to 13.5; and (*c*) Tables II and IV, *Groningen Publication* No. 27, corrected for scale error, $m = 4.0$ to 9.0. Systematic deviations in longitude and between northern and southern galactic hemispheres have been disregarded. The values of log N_m (N_m = number of stars per square degree brighter than m), extrapolated to $m = 21.0$, are in Tables XVII and XVIII. In general, the results refer to a grouping according to photographic magnitude, international scale. The means for latitudes 0°–90°, however, have been transformed into corresponding means for a grouping according to visual magnitude.

Total number of stars and galactic concentration.—To the twenty-first photographic magnitude the total number of stars is 890,000,000, or to the twentieth visual magnitude, 1,000,000,000. The values of dN_m/dm (Table XXII) from $m = 12$ or 13 to $m = 21$ can be represented by quadratic exponentials in m (Table XXIII). The assumption that these formulae also apply to the luminous stars beyond observational reach leads to 3×10^{10} as the total number of stars in the galactic system, 95 per cent of which are within 20° of the galactic plane. For the whole system, the ratio of N_m for latitudes 0° and 90° (galactic concentration) is more than 700. All results depending on the unrestricted summation of the quadratic exponentials are, however, very uncertain. Within the interval covered by the adopted distribution table, $m = 4.0$ to 21.0, the galactic concentration increases from 3.4 to 44 (Table XXV).

Mount Wilson Catalogue.—The *Catalogue* gives magnitudes for about 70,000 individual stars in Selected Areas 1–139, declinations +90° to −15°. Photographs with the 60-inch reflector, exposures 2^m and 15^m, measured at Mount Wilson, determine the scales and zero-points, and give magnitudes of 65,683 stars in fields 23′ in diameter. Plates of 60^m exposure, measured at Groningen, give the magnitudes of 44,910 stars in fields 15′ × 15′ or 20′ × 20′. The two series of measures were counted separately (Tables I*a*, I*b*), the counts being completed to a uniform limiting magnitude of 18.5. Cumulative sums, reduced to unit area, were formed for 28 latitude groups. Means and differences for Mount Wilson and Groningen counts are in Tables VII and VIII; smoothed values—the final result from the *Catalogue*—in Table IX.

Reduction of van Rhijn's distribution table to international scale.—For $m > 10$, Tables II and IV of *Groningen Publication* No. 27 depend on 65 Selected Areas in the *Harvard-Groningen Durchmusterung;* for $m < 6$, they are based on the visual magnitudes of *Harvard Annals*, **50**, corrected for color. The corresponding scale corrections from *Contributions* Nos. 288 and 289 are in Table XI, the interval $m = 6$ to 10 being bridged by interpolation.

Astrographic zones.—The published counts for 33 zones, mostly by Turner, were reduced to the international scale through the intermediary of *Groningen Publication* No. 27, corrected as above. Mean densities for 3–5 limiting magnitudes and latitudes 0°, 10°, 20° . . ., are given for each zone in Table XV; condensed means, in Table

[1] *Contributions from the Mount Wilson Observatory*, No. 301.

XVI. The latter, together with Table IX from the *Mount Wilson Catalogue*, is the basis of the adopted distribution (Table XVII) for $m > 9.0$.

Scale correction to Groningen Publication No. 18.—Table XX gives a mean scale correction for Kapteyn's distribution table, derived by comparison with $\log N_m$ in Table XVIII, 0°–90°, for a grouping according to visual magnitude. The correction seems to vary with the latitude.

The completion of the *Mount Wilson Catalogue of Selected Areas* and of certain preliminary investigations has made it possible to redetermine the distribution of stars with respect to apparent magnitude and galactic latitude. Counts of stars in this *Catalogue* cover the interval $m = 13.5$ to 18.5, international photographic scale. The distribution between $m = 9.0$ and 13.5 has been determined from 33 zones of the *Astrographic Catalogue* published by Turner.[1] These have been reduced to the international system through the intermediary of van Rhijn's Table IV, *Groningen Publication* No. 27, corrected for scale error. The revised distribution for $m < 9.0$ is that of van Rhijn's Table IV, extended to $m = 4.0$ by Table II of his paper and reduced to the international system. By what seems a permissible extrapolation, the results thus found have been extended to the twenty-first magnitude. The adopted values for the mean distribution are given, in the usual form, in Tables XVII and XVIII.

1. THE "MOUNT WILSON CATALOGUE"

Pending the appearance of the *Mount Wilson Catalogue*, much information relating to the observations and reductions and to the precision and consistency of the results may be found in the *Third Report on the Progress and Plan of Selected Areas*.[2] Certain details, however, are necessary for an understanding of the present discussion.

The *Catalogue* gives the photographic magnitudes of stars in Selected Areas 1–139, declinations $+90°$ to $-15°$. The photographs were made with the 60-inch reflector, in three series: (*a*) exposures of 2 minutes and 15 minutes, with the full aperture and with diaphragms and a wire-gauze screen; (*b*) exposures on the same plate of 60 minutes and 5 minutes, both with full aperture; (*c*) comparison exposures of 2 minutes, all with the full aperture, connecting each

[1] References are given in Table XIII.

[2] Groningen, 1923. See especially Second Appendix.

area with the adjacent areas of the same zone, and six areas in each zone with the North Pole.

Series (*a*) and (*c*) were measured and reduced at Mount Wilson, the diaphragm exposures being used to determine the scale in each area, while the comparison exposures served to reduce the results to the international zero-point. The fields are circular, 23′ in diameter, with the bright central star at the center of the plate. The results include magnitudes for 65,683 stars.

Series (*b*) was measured and reduced at Groningen with the aid of standard stars supplied from Mount Wilson, the secondary exposure of 5 minutes being used to extend the scale to the faint stars shown by the principal exposure of 60 minutes. To galactic latitude 40° the fields measured are 15′×15′; in higher latitudes they are 20′×20′. The centers are the even minutes of right ascension and the 10′ multiples of declination nearest to the positions of the central stars for 1900. Magnitudes were derived for 44,910 stars. The fields measured at Mount Wilson and Groningen thus overlap but do not coincide. Their respective areas are:

	Sq. Deg.
Mount Wilson	0.1154
Groningen, 0°–39°	0.0625
Groningen, 40°–90°	0.1111

The total number of individual stars is about 70,000.

The results agree well with the international scale.[1] It will be seen later that counts of the Groningen and Mount Wilson magnitudes are in excellent agreement, which establishes the identity of both scale and zero-point for the two series of measures, while it has already been shown[2] that the Mount Wilson magnitudes are on the international scale, at least to $m = 16$. Since the scales in the individual areas were established by methods used in deriving the standard magnitudes at the Pole, which are on the international system, it is reasonable to suppose that the accordance with this system extends to the limiting magnitude of the plates.

For stars occurring in both series of measures, the *Catalogue* gives the weighted mean magnitude; but in the initial stages of the

[1] *Transactions of the International Astronomical Union*, **1**, 69, 1922.

[2] *Mt. Wilson Contr.*, No. 289; *Astrophysical Journal*, **61**, 305, 1925.

following statistical discussion, it is both desirable and necessary to treat the Mount Wilson and Groningen results separately. The counts which are the basis of the discussion are given for each half-magnitude interval below $m = 13.0$ in Tables I*a* and I*b*, respectively. The numbers of stars brighter than $m = 13.0$ are listed in the second column; and in the last column but one, the total counted in each area. Thus for Area 1, the first line of Table I*a* indicates that the Mount Wilson counts in an area of 0.115 square degrees show a total of 4 stars brighter than 13.0; 0 stars in the interval 13.00–13.49; 1 in the interval 13.50–13.99, etc.; and a total of 137 for the area.

The numbers in the last columns of Tables I*a* and I*b* are the magnitude limits to which the counts are assumed to be complete. For a reflector having a focal ratio of 5 to 1, the limiting magnitude falls off rapidly with increasing distance from the optical axis. For the field used the maximum loss is about 0.5 mag., and allowance must be made for at least this amount in assigning the limit to which the counts are complete. For safety the limits have been rather conservatively fixed.

It should be explained that Table I*a* includes about 3800 faint stars which do not appear in the *Catalogue* because their magnitudes depend on a single plate. Series (*a*) includes two photographs of each area with two equal exposures of 15 minutes on each plate, but in the case of faint stars the magnitude for one of the plates is often missing. Defective and superposed images account for some of these cases, but difference in limiting magnitude is the usual explanation. Such stars, although retained for the statistical discussion, have been omitted from the *Catalogue*, unless measured at Groningen on plates of series (*b*). Everything that could be seen on the Mount Wilson photographs has been measured, and the relatively large error of measurement near the limiting magnitude explains the omissions from the *Catalogue*. The statistical value of the data is, however, little affected. All the Groningen magnitudes, on the other hand, depend on a single plate, but here there is less danger of error, for the measures could not be pushed so close to the limiting magnitude as in the case of the Mount Wilson plates where the multiple images guard against confusion with defects in the film.

TABLE I*a*

MOUNT WILSON COUNTS OF STARS IN SELECTED AREAS

Area	13.0	13		14		15		16		17		18		19		Total	Limit Complete
1	4	0	1	4	4	5	20	19	19	52	9	...	...	..	..	137	17.5
2	9	1	7	5	16	17	59	61	78	60	54	3	...	..	..	370	17.0
3	7	4	5	10	16	25	35	39	62	83	93	16	...	..	..	395	17.5
4	7	3	3	10	8	14	21	13	19	35	5	...	...	..	..	138	17.5
5	6	4	4	2	5	7	6	14	18	24	...	...	...	..	..	90	17.5
6	6	2	0	5	5	11	20	12	18	37	8	...	...	..	..	124	17.5
7	4	2	4	8	9	20	26	38	57	43	...	...	...	..	..	211	17.0
8	16	9	11	22	25	38	54	84	153	156	197	95	5	..	..	865	18.0
9	11	4	8	6	5	19	20	31	61	78	96	174	112	2	..	627	18.5
10	9	3	3	7	11	22	20	30	75	75	117	222	26	2	..	622	18.5
11	9	3	5	5	6	20	13	18	36	32	38	5	...	..	..	190	17.5
12	4	0	3	4	5	7	14	3	13	18	9	...	...	..	..	80	17.5
13	4	0	2	0	4	8	7	9	12	3	13	8	1	..	..	71	18.0
14	2	3	1	4	5	6	7	4	14	30	1	...	...	..	..	77	17.5
15	8	1	2	5	5	5	14	11	15	24	3	...	...	..	..	93	17.5
16	6	5	2	6	7	12	20	27	22	31	22	14	5	..	..	179	17.5
17	9	4	7	9	17	35	21	54	99	70	90	107	4	1	..	527	18.0
18	14	11	8	22	25	36	62	100	144	264	311	302	237	51	..	1587	18.0
19	20	11	17	16	24	39	45	57	113	102	170	258	20	..	..	892	18.5
20	13	11	9	4	13	34	36	42	74	70	81	89	267	16	..	759	18.0
21	17	4	4	13	17	33	34	30	132	66	127	241	48	2	..	768	18.5
22	10	10	8	19	19	29	46	58	81	136	128	132	153	17	1	847	18.5
23	10	5	7	16	15	22	29	53	129	141	122	305	200	17	..	1131	18.5
24	10	4	4	14	16	34	51	61	91	114	100	179	59	..	..	737	18.5
25	6	4	9	11	17	40	40	58	84	113	113	157	129	7	3	791	18.5

26	8	12	11	7	17	22	28	33	75	34	132	127	12	..	..	518	18.0
27	9	2	3	11	10	12	15	17	30	42	24	31	38	7	..	251	17.5
28	11	3	1	4	8	5	8	9	21	15	32	41	4	..	..	162	18.0
29	7	2	1	1	7	8	6	12	8	8	13	8	...	..	..	81	18.0
30	6	1	1	3	4	6	4	7	8	10	13	1	1	..	..	65	18.0
31	5	2	0	5	0	8	5	6	10	14	10	16	1	..	..	82	17.5
32	4	1	4	2	3	0	2	11	11	7	17	6	...	..	..	68	18.0
33	5	3	1	3	2	5	4	7	19	19	1	...	...	..	..	69	17.0
34	6	1	1	1	4	11	8	6	13	24	4	...	...	..	..	79	17.5
35	3	7	5	7	3	5	8	11	26	49	3	...	...	..	..	127	17.5
36	2	5	3	5	17	5	11	22	18	15	34	17	1	..	..	155	18.0
37	10	5	7	7	11	23	20	33	48	42	89	25	5	..	..	325	18.0
38	21	8	14	12	20	30	43	98	80	93	168	16	...	..	..	603	18.0
39	20	6	13	26	48	66	105	204	313	281	556	56	...	..	..	1694	18.0
40	18	6	14	17	23	60	58	66	219	239	259	213	47	2	..	1241	17.5
41	19	6	16	23	46	83	63	208	285	210	453	265	87	..	..	1764	18.0
42	10	4	6	20	36	30	54	151	88	110	265	238	20	..	..	1032	18.0
43	12	5	9	18	20	24	33	61	90	46	136	233	16	..	..	703	18.5
44	11	1	5	7	9	19	8	15	36	28	55	30	...	..	..	224	18.0
45	9	2	6	3	12	9	19	12	25	45	38	34	3	..	..	217	17.5
46	6	3	3	5	10	12	16	9	24	39	31	38	50	7	..	253	18.5
47	1	1	0	0	2	1	5	3	9	13	39	11	...	..	..	85	18.0
48	4	2	4	11	14	26	32	48	63	126	90	229	74	..	..	723	18.5
49	12	7	14	14	28	27	47	80	144	124	458	392	16	..	..	1363	18.0
50	16	10	14	19	29	29	79	81	127	176	172	219	200	42	4	1217	18.5
51	6	7	8	12	19	21	24	40	56	55	88	10	...	..	..	346	18.0
52	4	1	3	6	3	9	7	29	24	8	42	34	14	..	..	184	18.0
53	1	3	0	5	4	8	5	7	13	18	34	6	...	..	..	104	18.0
54	4	0	3	2	4	3	3	6	4	5	16	17	17	4	..	88	18.0
55	5	1	2	1	1	4	4	5	8	4	11	1	...	..	..	47	18.0

TABLE Ia—*Continued*

Area	13.0	13		14		15		16		17		18		19		Total	Limit Complete
56	4	1	4	2	1	6	2	8	13	12	17	1	...	..	..	71	18.0
57	2	2	3	4	5	4	4	5	3	11	10	4	3	23	12	95	18.0
58	6	1	2	5	7	3	4	7	7	12	30	13	5	..	..	102	18.0
59	3	2	3	7	5	0	8	20	11	19	50	18	1	..	..	147	18.0
60	7	2	2	2	10	6	14	20	20	18	25	7	1	..	..	134	18.0
61	11	3	6	7	15	9	25	39	30	31	53	54	16	1	1	301	18.0
62	17	11	3	12	20	20	24	87	57	100	222	118	24	11	..	726	18.0
63	17	12	21	21	66	84	71	135	380	251	217	251	154	49	..	1729	18.5
64	32	31	26	42	71	77	137	314	307	459	884	446	90	2	..	2918	18.0
65	15	9	12	20	24	47	75	98	215	241	257	526	146	1	..	1686	18.5
66	12	4	8	13	27	22	54	36	66	110	64	124	60	..	..	600	17.5
67	6	5	3	8	7	15	20	31	23	53	37	50	24	3	..	285	18.5
68	3	5	10	1	5	9	13	2	8	27	21	11	14	8	9	146	18.0
69	4	2	0	3	7	5	7	7	10	12	15	37	22	..	..	131	18.5
70	3	2	0	2	2	5	12	6	12	18	12	22	10	1	..	107	17.5
71	7	2	0	1	4	11	10	11	12	21	23	17	...	..	..	119	18.0
72	1	1	1	4	2	5	10	9	21	51	31	7	...	..	..	143	17.5
73	2	5	3	11	25	9	31	58	58	65	176	38	...	..	..	481	18.0
74	11	2	11	11	17	31	33	52	150	100	109	219	236	3	..	985	18.5
75	11	7	17	15	26	44	48	87	116	60	81	8	1	..	..	521	18.0
76	5	6	2	6	12	16	13	16	41	15	35	25	2	..	..	194	18.0
77	5	5	5	5	3	8	16	8	11	13	33	3	...	..	..	115	18.0
78	6	0	5	1	6	1	4	6	11	11	15	3	...	..	..	69	18.0
79	5	1	1	3	4	1	12	7	8	10	2	...	...	..	..	54	17.5
80	3	3	0	3	4	1	4	14	4	8	17	...	...	..	..	61	18.0
81	2	3	0	2	5	2	10	6	6	14	15	1	...	..	..	66	17.5
82	2	2	0	1	4	1	2	21	5	18	37	...	...	..	..	93	18.0
83	6	3	3	1	8	7	9	17	15	18	42	2	...	..	..	131	18.0
84	8	4	6	8	11	8	24	39	79	2	...	...	...	..	..	189	17.0
85	8	9	4	9	14	13	35	33	95	88	136	53	...	..	..	497	18.0

86	12	10	21	14	24	36	42	58	179	170	229	288	87	24	..	1194	18.5
87	10	3	10	12	9	27	46	66	88	292	244	432	405	25	1	1670	18.5
88	20	11	25	26	50	70	114	110	230	363	361	531	401	2	..	2314	18.5
89	10	6	8	4	17	19	28	77	63	69	162	165	31	..	..	659	18.0
90	8	2	4	6	6	17	14	14	34	35	36	38	24	43	18	299	18.0
91	4	0	1	5	7	7	7	21	21	15	28	84	13	..	..	213	18.5
92	4	2	4	1	2	7	8	10	4	7	13	14	13	9	..	98	18.5
93	9	1	1	6	5	5	6	4	6	8	11	26	4	..	..	92	18.5
94	10	2	1	3	3	2	8	12	11	11	13	31	9	1	..	117	18.5
95	2	0	1	1	3	7	10	2	20	12	16	26	1	..	..	101	18.5
96	7	2	6	6	9	13	24	23	42	30	25	47	12	1	..	247	18.5
97	6	2	5	3	8	14	20	36	54	63	78	98	12	1	..	400	18.5
98	38	8	13	34	56	40	116	151	189	559	155	59	17	..	..	1435	17.5
99	12	15	7	32	27	37	72	139	74	144	172	15	...	..	..	746	17.5
100	12	3	6	11	14	17	30	29	33	63	3	...	...	..	..	221	17.5
101	5	3	5	4	5	11	6	12	12	23	12	38	22	..	..	158	18.5
102	4	2	4	4	9	4	11	22	6	12	43	35	...	..	..	156	18.0
103	4	2	1	4	6	7	5	6	22	11	15	12	12	2	..	109	18.0
104	7	0	1	4	6	4	8	11	18	15	22	11	8	..	..	115	18.0
105	6	0	4	7	5	8	5	9	21	18	26	31	13	1	..	154	18.0
106	4	0	1	4	12	6	19	14	20	20	10	37	14	..	..	161	17.5
107	7	5	3	3	3	14	6	7	27	56	54	21	4	..	..	210	18.0
108	7	2	3	6	10	22	23	64	84	65	100	69	...	..	..	455	18.0
109	6	3	13	4	11	36	43	59	124	233	162	273	217	22	..	1206	18.5
110	7	1	1	2	2	6	4	4	20	14	16	59	11	..	..	147	18.5
111	4	5	10	7	19	43	73	79	131	360	361	383	279	20	..	1774	18.5
112	8	5	9	12	20	34	37	27	99	85	170	183	4	..	..	693	18.0
113	9	3	2	4	17	15	6	19	35	21	27	87	43	7	..	295	18.5
114	3	3	4	3	10	11	8	11	21	41	44	10	...	..	..	169	17.5
115	5	3	2	2	2	6	6	9	12	9	15	25	16	..	..	112	18.5

TABLE Ia—*Continued*

Area	13.0	13		14		15		16		17		18		19		Total	Limit Complete
116	6	1	1	2	3	4	2	3	5	9	7	6	17	7	8	81	18.0
117	1	2	3	1	5	2	2	7	16	5	12	23	1	..	..	80	18.5
118	4	1	2	0	3	3	6	8	8	3	41	4	...	..	..	83	18.0
119	3	4	6	2	5	4	6	7	12	10	11	10	9	..	..	89	18.0
120	6	3	1	5	5	6	6	13	29	22	37	11	...	..	..	144	18.0
121	2	5	5	3	14	11	17	28	23	34	36	41	14	..	..	233	17.5
122	8	6	6	14	18	23	34	81	69	109	188	36	...	..	..	592	18.0
123	36	22	14	47	81	76	111	252	232	288	399	134	20	..	..	1712	18.0
124	18	4	20	21	29	53	43	173	103	104	270	21	1	..	..	860	18.0
125	13	6	7	5	21	22	22	47	27	64	56	9	...	..	..	299	17.5
126	6	3	3	6	8	8	10	15	31	25	43	40	3	..	..	201	18.0
127	6	2	2	5	8	10	9	17	15	45	27	1	...	..	..	147	17.5
128	7	1	4	4	2	9	6	19	21	14	43	10	...	..	..	140	18.0
129	5	3	4	6	9	10	4	14	18	28	26	26	5	..	..	158	17.5
130	6	2	2	4	7	11	7	14	39	27	53	34	6	..	..	212	18.0
131	9	4	1	2	7	16	16	34	37	46	25	12	1	..	..	210	17.5
132	6	3	2	6	5	6	16	37	67	50	89	73	6	..	..	366	18.0
133	5	3	6	8	14	17	35	95	167	233	293	106	19	4	..	1005	18.0
134	35	11	22	24	31	70	95	171	132	172	314	49	...	..	..	1126	18.0
135	15	13	11	27	49	54	135	203	259	648	705	...	...	..	..	2119	17.5
136	10	6	5	10	20	29	28	65	66	159	180	3	...	..	..	581	17.5
137	6	4	4	6	5	9	13	20	27	37	59	52	...	..	..	242	18.0
138	4	4	3	3	2	10	7	11	19	17	15	25	7	..	..	127	18.5
139	3	1	3	3	1	6	7	6	9	15	16	9	2	1	..	82	18.0
Sums	1170	588	780	1162	1859	2593	3617	5867	8439	10,612	13,424	10,602	4464	449	57	65,683	.....

TABLE Ib

GRONINGEN COUNTS OF STARS IN SELECTED AREAS

Area	13.0	13		14		15		16		17		18		19		Total	Limit Complete
1	4	0	1	2	2	3	9	8	12	33	34	8	...	..	..	116	17.5
2	3	0	2	1	10	10	40	26	37	46	96	34	52	3	..	360	18.0
3	4	4	4	4	9	10	16	15	39	40	63	43	5	..	..	256	18.0
4	4	1	2	4	5	7	11	10	12	15	16	...	...	..	..	87	17.5
5	6	5	4	2	3	7	6	13	19	32	22	2	...	..	..	121	17.5
6	2	2	0	3	5	4	11	10	7	24	11	12	...	..	..	91	17.5
7	3	1	2	4	6	10	15	20	32	33	24	66	15	..	..	231	17.0
8	8	3	5	10	15	21	25	38	63	71	129	157	130	..	..	675	18.5
9	4	3	4	1	3	13	11	16	36	51	61	86	58	..	..	347	18.5
10	6	3	1	3	7	7	10	18	38	54	37	28	...	..	..	212	17.5
11	4	2	2	2	3	11	10	13	20	20	19	23	18	30	1	178	18.5
12	5	0	2	5	6	6	12	5	9	12	10	7	10	..	..	89	17.5
13	4	0	2	0	3	6	7	8	10	12	41	7	...	..	..	100	18.0
14	1	3	1	4	4	7	6	4	12	33	15	12	...	..	..	102	17.5
15	8	0	2	4	6	7	16	10	15	41	24	18	...	..	..	151	17.5
16	3	4	2	4	4	7	11	16	14	20	15	11	1	..	..	112	17.5
17	6	4	2	2	11	13	9	20	41	54	46	71	13	..	..	292	18.5
18	9	8	3	10	14	15	33	60	84	135	194	148	100	3	..	816	18.0
19	11	8	10	10	10	26	28	27	61	66	77	90	23	..	..	447	18.0
20	8	8	4	1	7	19	25	35	32	36	49	32	50	6	..	312	18.0
21	7	3	3	6	11	18	15	17	75	39	70	80	4	..	..	348	18.0
22	6	6	3	10	11	17	22	30	53	90	74	71	50	5	..	448	17.5
23	6	1	3	9	8	14	15	34	62	86	65	90	49	3	..	445	18.5
24	6	3	2	8	6	15	36	31	45	88	55	139	47	..	..	481	18.5
25	3	1	3	3	11	26	25	31	42	45	64	80	55	..	..	389	18.0

TABLE I*b*—*Continued*

Area	13.0	13	14	15	16	17	18	19	Total	Limit Complete
26	4	8 4	3 9	10 12	20 35	26 39	91 24	30 ..	315	18.5
27	7	0 2	6 6	5 8	10 17	21 16	21 27	2 ..	148	18.5
28	6	3 0	3 5	2 3	4 19	4 9	15 6	.. ..	79	18.5
29	8	1 1	2 5	8 7	11 6	12 18	7 ...	.. ..	86	18.0
30	5	1 1	4 3	5 4	7 8	6 17	3 ...	.. ..	64	18.0
31	6	2 0	4 1	8 3	6 10	13 16	13 ...	.. ..	82	18.0
32	4	1 2	3 2	1 4	12 11	6 13	12 1	.. ..	72	18.0
33	5	3 1	3 1	5 3	8 19	28 11	... ...	.. ..	87	17.5
34	7	1 1	1 4	10 6	9 15	24 6	1 ...	.. ..	85	17.5
35	4	8 3	7 3	4 8	11 27	55 34	... ...	.. ..	164	17.5
36	1	4 3	2 9	2 5	11 12	6 23	19 6	.. ..	103	18.0
37	5	5 5	5 6	13 13	21 20	32 46	39 44	.. ..	254	18.0
38	12	5 7	8 9	16 24	39 50	71 77	31 1	.. ..	350	17.5
39	10	3 5	13 28	37 54	95 210	131 341	154 ...	.. ..	1081	18.0
40	9	3 8	7 12	27 38	39 97	147 170	201 199	.. ..	957	18.0
41	10	2 11	10 17	41 67	77 102	160 307	191 62	.. ..	1057	18.0
42	8	3 3	12 13	13 32	43 51	127 91	58 3	.. ..	457	17.5
43	3	6 7	8 12	19 14	25 29	66 61	40 13	.. ..	303	18.0
44	7	1 1	7 5	11 5	12 9	16 29	9 8	.. ..	120	18.0
45	5	2 4	4 3	8 7	12 7	24 13	21 45	.. ..	155	19.0
46	4	1 2	2 2	8 8	12 14	14 17	14 27	30 7	162	19.0
47	1	0 0	0 2	0 3	3 3	7 9	26 21	.. ..	75	18.5
48	4	0 5	3 8	8 19	23 40	54 56	125 131	12 ..	488	18.5
49	7	3 8	11 13	14 29	41 68	84 172	308 106	131 ..	995	18.5
50	7	2 6	15 15	13 31	73 71	106 79	116 167	18 ..	719	18.5
51	3	5 4	4 17	13 9	23 29	24 31	46 ...	.. ..	208	18.5
52	3	0 0	6 3	3 8	12 6	6 17	13 15	.. ..	92	18.5
53	2	3 0	6 4	4 10	11 8	18 15	31 26	11 ..	149	18.5
54	2	1 3	2 7	1 5	5 1	8 9	18 23	20 1	106	19.0
55	4	1 2	1 1	5 3	6 3	8 9	6 ...	.. ..	49	18.0

56	4	0	4	1	3	1	10	7	9	9	12	33	...	..	..	93	18.5
57	2	2	3	5	8	2	4	3	8	4	11	8	10	22	8	100	19.0
58	5	1	1	5	7	3	2	3	9	16	23	22	13	1	..	111	18.0
59	3	1	3	6	5	5	7	11	8	20	55	53	2	..	..	179	18.0
60	7	2	2	2	8	7	13	19	19	26	28	13	...	..	..	146	17.5
61	5	2	5	5	6	5	17	8	15	35	18	27	6	..	..	154	18.5
62	11	6	3	5	11	8	19	34	46	34	117	59	11	..	..	364	18.0
63	8	9	10	18	26	49	70	64	126	113	264	139	134	..	..	1030	18.0
64	20	19	8	27	41	43	91	149	160	241	404	347	174	..	..	1724	18.0
65	12	2	5	14	12	25	49	41	134	126	185	297	148	106	..	1156	18.5
66	6	1	1	5	14	22	23	16	26	26	101	73	33	..	..	347	18.5
67	5	2	1	5	3	9	12	16	10	25	22	30	29	30	4	203	18.5
68	3	2	9	1	5	9	10	3	13	23	26	18	13	7	2	144	18.0
69	4	2	1	2	6	6	9	4	10	11	17	27	14	1	..	114	18.5
70	3	0	0	2	2	4	11	6	13	17	17	23	4	..	..	102	18.0
71	6	1	0	1	2	5	3	6	10	8	16	21	5	..	..	84	18.5
72	1	1	0	3	0	3	5	5	12	20	86	5	...	..	..	141	18.0
73	1	1	2	9	12	6	19	31	30	48	114	102	3	..	..	378	18.0
74	6	1	8	7	10	19	17	25	96	47	68	98	132	5	..	539	18.5
75	8	5	9	8	13	23	28	44	53	42	80	47	53	20	..	433	18.0
76	4	5	2	4	7	9	10	5	29	13	29	43	27	26	..	213	18.5
77	3	2	4	3	1	4	9	7	3	7	27	17	...	..	..	87	18.5
78	6	0	3	1	6	2	4	6	12	13	28	6	...	..	..	87	18.0
79	5	1	1	3	5	2	9	6	11	8	21	12	21	2	..	107	18.0
80	3	3	1	2	3	1	7	11	5	10	20	16	27	2	..	111	18.0
81	3	3	1	3	4	3	8	5	8	16	17	13	16	35	10	145	18.0
82	2	2	0	1	3	3	1	17	7	17	44	40	6	..	..	143	18.0
83	5	3	3	1	7	5	9	14	15	18	38	29	21	..	..	168	18.0
84	8	4	4	8	11	9	26	34	36	57	53	70	21	..	..	341	18.5
85	4	4	0	6	7	7	18	19	51	63	72	18	...	..	..	269	17.5

TABLE I*b*—*Continued*

Area	13.0	13	14	15	16	17	18	19	Total	Limit Complete
86	4	4 13	9 15	25 20	29 89	109 121	156 36	.. ..	630	18.5
87	5	1 8	9 3	19 28	37 48	152 190	255 213	.. ..	968	18.5
88	10	9 13	11 23	40 53	62 117	176 218	343 418	66 ..	1559	18.5
89	8	0 4	1 7	11 20	34 43	33 73	81 36	2 ..	353	18.5
90	6	2 2	6 2	10 6	10 13	19 21	22 20	11 4	154	18.5
91	5	0 1	4 8	6 9	16 23	19 31	55 12	.. ..	189	18.5
92	5	2 3	1 3	8 10	11 1	13 35	28 38	16 ..	174	18.5
93	10	1 3	6 5	5 3	7 6	11 20	23 3	.. ..	103	18.0
94	10	2 1	3 3	1 7	9 9	9 13	15 7	.. ..	89	18.0
95	1	0 0	1 3	5 5	1 7	4 8	13 ...	.. ..	48	18.5
96	3	0 2	4 4	9 14	7 22	22 18	34 26	16 24	205	18.5
97	5	1 2	1 4	4 10	20 24	30 33	68 126	18 ..	346	18.5
98	21	5 8	20 36	22 56	96 115	281 222	22 ...	.. ..	904	17.5
99	6	9 4	24 16	20 31	71 44	79 82	13 ...	.. ..	399	17.5
100	10	2 3	6 8	8 14	11 15	39 4	... ...	.. ..	120	17.5
101	4	3 6	4 4	10 6	12 10	19 13	31 41	52 24	239	19.0
102	6	2 4	4 5	4 9	18 10	13 34	32 ...	.. ..	141	18.0
103	4	2 0	3 6	7 5	6 22	12 13	31 28	.. ..	139	18.5
104	6	0 0	4 6	4 9	12 17	19 26	8 ...	.. ..	111	18.0
105	6	0 4	6 4	7 5	12 17	20 29	40 11	.. ..	161	18.5
106	4	0 1	1 11	7 17	13 20	22 11	7 ...	.. ..	114	17.5
107	7	6 4	3 2	15 5	8 21	54 57	13 ...	.. ..	195	18.0
108	3	1 1	5 6	17 10	42 45	53 68	62 ...	.. ..	313	18.0
109	2	2 7	2 5	19 25	31 62	122 103	143 23	.. ..	546	18.5
110	6	1 1	1 0	4 4	4 6	7 7	30 14	19 3	107	18.5
111	2	2 7	4 15	24 37	52 70	195 245	196 226	17 ..	1092	18.0
112	5	3 6	11 10	18 21	12 40	57 75	72 3	.. ..	333	18.0
113	3	2 0	1 8	9 3	8 12	16 22	28 18	.. ..	130	18.5
114	2	3 4	4 8	10 7	10 22	24 64	43 8	.. ..	209	18.0
115	7	2 2	2 2	6 5	10 12	5 21	23 18	2 ..	117	18.5

116	4	1	1	3	3	5	3	3	4	6	6	12	15	7	1	74	19.0
117	1	2	4	1	5	2	4	5	15	10	17	24	2	..	..	92	18.5
118	5	1	1	0	4	4	7	6	8	22	42	7	...	..	..	107	18.0
119	2	4	8	2	5	2	7	5	10	14	13	7	1	..	..	80	17.5
120	5	2	2	3	5	6	8	11	30	21	40	34	4	..	..	171	18.0
121	2	3	3	2	9	4	7	15	11	20	26	31	12	..	..	145	18.0
122	6	3	5	7	11	12	23	44	48	61	138	71	...	..	..	429	18.0
123	24	14	8	28	44	43	58	127	167	174	256	329	16	..	..	1288	18.5
124	10	4	17	15	17	26	26	100	68	69	94	...	...	..	..	446	18.0
125	11	4	2	3	12	14	12	22	27	28	41	30	2	..	..	208	18.0
126	4	2	1	2	8	4	5	7	18	15	15	25	12	2	..	120	18.5
127	7	1	2	6	6	10	7	17	17	38	27	1	...	..	..	139	17.5
128	7	2	4	4	3	11	4	16	21	18	43	12	...	..	..	145	18.0
129	4	2	4	3	10	8	5	13	18	27	25	39	2	..	..	160	18.5
130	5	3	2	5	7	12	7	17	35	25	48	35	2	..	..	203	18.0
131	5	3	0	2	3	5	10	16	19	22	53	39	36	1	..	214	18.0
132	4	3	1	3	3	5	9	23	31	33	54	39	...	..	..	208	18.0
133	1	1	4	3	11	13	19	55	105	133	215	209	418	118	..	1305	19.0
134	24	6	16	15	17	42	57	99	101	117	221	169	...	..	..	884	18.0
135	9	4	7	18	32	31	70	140	176	423	819	300	...	..	..	2029	18.0
136	6	2	3	8	11	18	22	44	33	100	116	...	...	..	..	363	17.5
137	5	0	3	4	4	6	12	17	25	23	35	39	17	..	..	190	18.0
138	5	3	5	4	4	8	9	12	26	28	30	29	1	..	..	164	17.5
139	3	1	3	2	1	6	4	5	10	13	18	11	...	..	..	77	18.0
Sums	792	376	482	741	1120	1563	2235	3305	4836	6667	9344	8029	4423	908	89	44910	.....

2. DISCUSSION OF THE MOUNT WILSON COUNTS

The data in Tables I*a* and I*b* lead to the values of log N_m for different galactic latitudes given in Table VII, where N_m is the mean number of stars per square degree brighter than m derived from the two series of counts. These results, which are the basis for the mean distribution for $m > 13.5$, can be reproduced from the original counts by following the steps now to be described.

Formation of latitude groups.—Since the fields measured are small, the counts are much affected by local irregularities of distribution. These are partially eliminated by combining the 139 areas into 28 groups, arranged according to galactic latitude as shown in Table II, each line of which gives in succession the mean latitude of the group, the serial numbers of the areas, and their respective latitudes. Southern latitudes are indicated by italic figures, although in the present discussion any possible systematic difference between the two galactic hemispheres is ignored. For the completion of the counts to a uniform limiting magnitude, the data were further condensed into seven groups indicated by the divisions of Table II.

Completion of the counts.—For the discussion it is important that the counts for all areas should be complete to the same limiting magnitude. The frequencies of the limits of completeness given in the last columns of Tables I*a* and I*b* are shown in Table III. For a considerable percentage of cases, the data are complete to magnitude 18.5; for the others, the distribution is such that the counts can be extended to this limit with little uncertainty.

The procedure was as follows: The original counts in Tables I*a* and I*b* were tabulated in subgroups according to the latitude and magnitude limits shown in Table III. Thus for Mount Wilson, group I, the tabulation included three subgroups: two areas complete to 17.5, eight complete to 18.0, and nine complete to 18.5. The logarithms of the sums of the numbers of stars in each half-magnitude interval were plotted for each subgroup. The ordinates of the resulting curves (log S curves) were tabulated for each half-magnitude and differenced, the individual first differences thus having the form

$$\log R = \log \left(\frac{\text{No. stars } m \text{ to } m+0.5}{\text{No. stars } m-0.5 \text{ to } m} \right)$$

where, briefly, R is the ratio of the numbers of stars in adjacent half-magnitude intervals. Each subgroup thus gives a sequence of values of log R, terminating at $m = 17.0$, 17.5, or 18.0, and, for a few scattering areas, at 16.5. Means were formed for corresponding

TABLE II

GROUPING OF AREAS ACCORDING TO GALACTIC LATITUDE

Group	Mean Lat.	Serial Numbers of Areas					Galactic Latitudes				
I	0°	24,	40,	123,	134,	...	0°,	0°,	0°,	0°,	..
	1	19,	64,	74,	87,	98	*1*,†	*1*,	1,	1,	1
	3	(110),	9,	49,	8,	18	1,	2,	*2*,	*3*,	6
	8	23,	25,	41,	39,	50	*7*,	8,	*8*,	9,	10
II	11	63,	97,	111,	48,	65	10,	*11*,	*11*,	*12*,	*12*
	12	73,	88,	124,	133,	2	*12*,	*12*,	12,	12,	13
	13	10,	22,	42,	122,	135	13,	*13*,	*13*,	*13*,	*13*
	15	75,	109,	86,	99,	43	14,	14,	15,	15,	*16*
III	18	3,	21,	20,	26,	38	17,	*17*,	*18*,	18,	18
	20	17,	7,	66,	47,	51	19,	20,	*20*,	*21*,	22
	24	62,	89,	72,	125,	132	23,	*23*,	*24*,	24,	24
	26	96,	112,	11,	121,	136	*25*,	*25*,	26,	*26*,	*26*
IV	27	46,	76,	85,	1,	27	*27*,	27,	27,	28,	28
	29	37,	67,	100,	108,	4	28,	*28*,	28,	28,	32
	33	44,	45,	16,	52,	61	*32*,	*32*,	33,	34,	34
	35	71,	90,	131,	126,	6	*34*,	*34*,	34,	35,	36
V	39	95,	113,	28,	36,	77	*38*,	*38*,	39,	39,	39
	40	137,*	84,	120,	12,	101	*39*,*	40,	*40*,	41,	41
	42	107,	5,	70,	127,	91	41,	42,	*42*,	42,	*43*
	46	130,	53,	68,	69,	128	43,	47,	*47*,	*47*,	47
VI	48	129,	15,	60,	29,	35	47,	48,	48,	49,	49
	51	94,	114,	106,	102,	13	*49*,	*49*,	51,	52,	53
	54	138,	78,	83,	119,	14	*53*,	54,	54,	*54*,	57
	59	93,	115,	34,	59,	30	*58*,	*58*,	59,	59,	60
VII	61	54,	103,	105,	92,	104	60,	60,	60,	*62*,	62
	66	118,	82,	139,	79,	31	*65*,	66,	*66*,	67,	68
	73	33,	32,	58,	55,	80	68,	72,	73,	74,	76
	79	81,	116,	117,	56,	57	76,	*76*,	*76*,	80,	85

* Areas of Groningen fields 15′×15′ for latitudes less than 40°; 20′×20′ for higher latitudes.

† Italic figures indicate negative latitudes. The position used for the galactic pole is $\alpha = 12^h\ 41^m\ 20^s$, $\delta = +27°\ 21'$ (1875).

intervals, Mount Wilson and Groningen results and each of groups I–VII being treated separately. Finally, the Mount Wilson and Groningen means, the agreement of which affords a useful check, were themselves combined to form for each latitude group a single mean sequence of values of log R. The values actually required for the completion of the counts are given in Table IV.

The values of log R, being first differences of the ordinates of the log S curves, can be used to extend the curves terminating at 17.0, 17.5, and 18.0 to the uniform limit of 18.5. Thus again for Mount

TABLE III

FREQUENCIES OF LIMITS TO WHICH ORIGINAL COUNTS ARE COMPLETE

GROUP	GAL. LATITUDE	MOUNT WILSON				GRONINGEN				
		17.0	17.5	18.0	18.5	17.0	17.5	18.0	18.5	19.0
I	0°–10°	0	2	8	9	0	1	8	10	0
II	10 –16	1	2	6	11	0	4	9	6	1
III	17 –26	1	7	10	2	1	2	9	8	0
IV	27 –36	0	8	10	2	0	6	4	8	2
V	38 –47	1	4	10	5	0	3	8	8	1
VI	47 –60	0	7	9	4	0	8	10	2	0
VII	60 –85	1	3	14	2	0	1	11	5	3
Totals	.....	4	33	67	35	1	25	59	47	7

Wilson, group I, the last ordinate of the log S curve for the eight areas complete to 18.0 is 3.53, which is the logarithm of the total number of stars in these areas having magnitudes between 17.5 and

TABLE IV

VALUES OF LOG R USED TO COMPLETE COUNTS

GROUP	m		
	17.0	17.5	18.0
I	0.151	0.141	0.130
II	.160	.149	.138
III	.138	.130	.120
IV	.122	.111	.105
V	.116	.107	.096
VI	.109	.100	.092
VII	0.108	0.100	0.093

18.0. Applying now log $R=0.13$, from Table IV, we find for the interval 18.0–18.5, log $S=3.66$. The total number of stars in the eight areas having magnitudes in this interval is therefore 4570, which is the "completed" total for this particular subgroup of areas. These stars are now to be distributed among the individual areas

in accordance with the totals counted in each area to the "observed" limit of completeness, namely, 18.0. The details are as follows:

S.A. No.....	123	134	64	49	8	18	41	39	Total
$N_{18.0}$.....	1558	1077	2380	955	765	997	1412	1638	10782
Fraction.....	0.145	0.100	0.220	0.089	0.071	0.092	0.131	0.152	1.000
$N_{18.0-18.5}$.....	663	457	1005	407	324	420	599	695	4570

The numbers counted in each area to 18.0, as found from Table I*a*, are given in the second line, while their fractional distribution among the total of 10,782 appears in the third line. The numbers in the last line give the corresponding distribution of the 4570 stars in the interval 18.0–18.5. These are the completed counts for the individual areas, and together with similar results for other areas are collected in Table V.

Systematic correction for central stars.—The original counts require a correction arising from the fact that each field is centered on a bright star, usually of the eighth or ninth magnitude. The areas measured are, however, so small that a random choice of fields would by no means always include a star of this order of brightness. For example, suppose all the central stars were between magnitudes 8.5 and 9.0, and that all the fields were exactly $\frac{1}{9}$ square degree. Counts of the central stars alone, on the assumption that the fields were representative, would then give 9 stars per square degree between 8.5 and 9.0, whereas the actual number for all latitudes averages less than a single star. The counts for the interval in question and all the cumulative sums to lower limits of magnitude would be too large by about 8 stars per square degree. The percentage error for faint magnitudes, especially in rich regions, is small, but this is by no means the case in general. Thus to $m = 15.0$, in latitude 70°, the average total number of stars per square degree is about 100, and the systematic error would accordingly be 8 per cent.

The required corrections have been calculated for each area on the basis of the adopted mean magnitude of the central star and the distribution table of van Rhijn, *Groningen Publication* No. 27, corrected for scale, which, for the region of brightness involved, is practically identical with Table XVII, the final result of the present discussion. The corrections, referred to an area of 1 square degree, are in the last two columns of Table V. The magnitudes of the

TABLE V

COMPLETED COUNTS AND CORRECTIONS FOR CENTRAL STARS

S.A.	M.W.		Gr.		Mag. Cent. Star	Gal. Lat.	Corr. for Cent. Star	
	17.5–18.0	18.0–18.5	17.5–18.0	18.0–18.5			M.W.	Gr.
1....	48	62	23	30	7.27	+28°	8.5	15.9
2*...	195	275	.....	118	9.95	+13	6.4	13.8
3....	111	146	.....	75	8.51	+17	8.0	15.3
4....	50	64	22	29	9.94	+32	7.4	14.7
5....	31	38	28	36	9.35	+42	8.0	8.4
6....	43	56	21	28	10.49	+36	6.9	14.2
7*...	102	135	58	76	10.23	+20	5.9	13.2
8....	.....	324	.....	.....	8.84	− 3	7.6	14.9
9....	.....	.....	.....	.....	7.63	+ 2	8.3	15.6
10....	.....	.....	70	99	8.89	+13	7.8	15.2
11....	57	76	.....	.....	8.30	+26	8.3	15.7
12....	24	30	18	23	8.64	+41	8.2	8.6
13....	.....	16	.....	25	7.83	+53	8.5	8.9
14....	24	29	22	27	10.50	+57	6.6	6.9
15....	28	35	32	40	9.98	+48	7.7	8.0
16....	51	66	27	34	9.16	+33	7.9	15.2
17....	.....	161	.....	.....	9.05	+19	7.6	14.9
18....	.....	420	.....	225	10.08	+ 6	4.1	11.4
19....	.....	.....	.....	133	9.36	− 1	6.8	14.2
20....	.....	150	.....	81	8.97	−18	8.0	15.3
21....	.....	.....	.....	95	10.03	−17	5.6	13.0
22....	.....	.....	119	168	8.77	−13	7.8	15.2
23....	.....	.....	.....	.....	8.83	− 7	7.7	15.0
24....	.....	.....	.....	.....	10.28	0	3.8	11.2
25....	.....	.....	.....	101	9.29	+ 8	7.1	14.4
26....	.....	147	.....	.....	10.47	+18	5.7	13.0
27....	56	72	.....	.....	10.40	+28	6.4	13.7
28....	.....	37	.....	.....	10.59	+39	5.9	13.3
29....	.....	19	.....	21	8.78	+49	8.3	8.6
30....	.....	16	.....	16	10.4†	+60	7.4	7.7
31....	16	19	.....	20	9.49	+68	8.2	8.5
32....	.....	18	.....	17	9.71	+72	7.9	8.3
33*...	18	22	26	32	9.40	+68	8.2	8.5
34....	24	29	23	28	9.63	+59	7.8	8.1
35....	39	48	39	47	10.85	+49	6.3	6.6
36....	.....	43	.....	23	9.34	+39	8.0	15.3
37....	.....	98	.....	48	10.58	+28	5.0	12.3
38....	.....	228	92	122	9.71	+18	6.8	14.1
39....	.....	695	.....	370	9.16	+ 9	7.2	14.5
40....	297	401	.....	223	9.84	0	5.6	12.9
41....	.....	599	.....	321	10.06	− 8	4.4	11.7
42....	.....	338	146	206	8.3†	−13	8.2	15.5
43....	.....	.....	.....	110	10.10	−16	5.5	12.8
44....	.....	64	.....	29	8.96	−32	8.2	15.5
45....	53	68	.....	.....	9.00	−32	7.9	15.2

* Completed counts for 17.0–17.5:

S.A.	M.W.	Gr.
2	141	..
7	76	43
33	14	..
84	63	..

† Combined brightness of components.

TABLE V—*Continued*

S.A.	M.W.		Gr.		Mag. Cent. Star	Gal. Lat.	Corr. for Cent. Star	
	17.5–18.0	18.0–18.5	17.5–18.0	18.0–18.5			M.W.	Gr.
46	.....	.....	.....	.....	9.51	−27°	7.2	14.6
47	.....	28	.....	.....	7.46	−21	8.5	15.9
48	.....	.....	.....	.....	(6.9)‡	−12	.....	.....
49	.....	407	.....	.....	9.90	− 2	5.8	13.2
50	.....	.....	.....	.....	8.55	+10	7.8	15.1
51	.....	130	.....	.....	10.71	+22	4.6	12.0
52	.....	46	.....	.....	8.53	+34	8.2	15.5
53	.....	31	.....	.....	9.61	+47	7.6	8.0
54	.....	14	.....	.....	10.23	+60	7.4	7.7
55	.....	13	.....	12	10.08	+74	7.5	7.9
56	.....	20	.....	.....	10.13	+80	7.6	8.0
57	.....	15	.....	.....	9.01	+85	8.2	8.6
58	.....	24	.....	21	9.72	+73	8.0	8.3
59	.....	33	.....	33	9.42	+59	8.1	8.5
60	.....	32	31	38	9.50	+48	7.7	8.0
61	.....	76	.....	.....	9.56	+34	7.4	14.7
62	.....	222	.....	106	8.36	+23	8.3	15.6
63	.....	.....	.....	328	9.61	+10	6.2	13.6
64	.....	1005	.....	479	8.31	− 1	8.0	15.3
65	.....	.....	.....	.....	9.45	−12	7.3	14.7
66	130	180	.....	.....	10.10	−20	5.9	13.2
67	.....	.....	.....	.....	9.67	−28	7.3	14.6
68	.....	33	.....	31	9.08	−47	8.0	8.4
69	.....	.....	.....	.....	8.22	−47	8.4	8.8
70	21	26	.....	23	8.20	−42	8.4	8.8
71	.....	34	.....	.....	9.74	−34	7.4	14.7
72	41	54	.....	49	6.89	−24	8.6	15.9
73	.....	193	.....	118	8.34	−12	8.1	15.5
74	.....	.....	.....	.....	8.89	+ 1	7.6	14.9
75	.....	223	.....	136	9.62	+14	6.5	13.9
76	.....	55	.....	.....	9.57	+27	7.2	14.6
77	.....	35	.....	.....	8.66	+39	8.2	15.6
78	.....	17	.....	22	9.58	+54	7.8	8.1
79	15	18	.....	21	9.60	+67	7.9	8.2
80	.....	18	.....	19	8.53	+76	8.4	8.7
81	14	17	.....	20	10.29	+76	7.6	7.9
82	.....	27	.....	28	9.74	+66	7.9	8.3
83	.....	33	.....	31	9.25	+54	8.1	8.4
84*	79	100	.....	.....	8.82	+40	8.2	8.6
85	.....	148	56	72	9.37	+27	7.6	15.1
86	.....	.....	.....	.....	10.49	+15	5.4	12.7
87	.....	.....	.....	.....	9.66	+ 1	5.6	13.0
88	.....	.....	.....	.....	8.82	−12	7.8	15.2
89	.....	180	.....	.....	8.70	−23	8.0	15.4
90	.....	58	.....	.....	9.04	−34	7.9	15.2
91	.....	.....	.....	.....	8.86	−43	8.2	8.6
92	.....	.....	.....	.....	9.24	−62	8.1	8.5
93	.....	.....	.....	21	10.10	−58	7.3	7.7
94	.....	.....	.....	18	10.58	−49	6.3	6.6
95	.....	.....	.....	.....	9.34	−38	8.0	15.3
96	.....	.....	.....	.....	9.46	−25	7.8	15.1

‡ Approximate magnitude. Too bright for measurement.

TABLE V—*Continued*

S.A.	M.W.		Gr.		Mag. Cent. Star	Gal. Lat.	Corr. for Cent. Star	
	17.5–18.0	18.0–18.5	17.5–18.0	18.0–18.5			M.W.	Gr.
97	.....	.....	.....	.....	6.81	−11°	8.6	15.8
98	497	670	269	363	7.92	+ 1	8.3	15.6
99	358	505	146	206	9.81	+15	6.6	13.9
100	81	105	36	47	9.27	+28	7.8	15.1
101	.....	.....	.....	.....	9.42	+41	8.0	8.3
102	.....	31	.....	29	9.12	+52	8.1	8.4
103	.....	24	.....	.....	8.94	+60	8.3	8.7
104	.....	28	.....	29	10.89	+62	6.8	7.1
105	.....	31	.....	.....	9.74	+60	7.8	8.2
106	31	39	28	35	10.65	+51	6.5	6.8
107	.....	58	.....	54	8.04	+41	8.4	8.8
108	.....	128	.....	70	9.37	+28	7.8	15.1
109	.....	.....	.....	.....	9.08	+14	7.4	14.8
110	.....	.....	.....	.....	10.31	+ 1	4.0	11.3
111	.....	.....	.....	283	(7.0)‡	−11	.....	.....
112	.....	195	.....	93	9.56	−25	7.2	14.5
113	.....	.....	.....	.....	9.33	−38	8.0	15.3
114	36	44	.....	42	9.64	−49	7.7	8.1
115	.....	.....	.....	.....	9.66	−58	7.8	8.2
116	.....	12	.....	.....	10.28	−76	7.6	7.9
117	.....	.....	.....	.....	10.21	−76	7.6	7.9
118	.....	23	.....	29	8.96	−65	8.4	8.7
119	.....	18	17	21	9.13	−54	8.1	8.4
120	.....	42	.....	40	10.22	−40	6.9	7.2
121	56	73	.....	37	8.41	−26	8.3	15.7
122	.....	242	.....	155	9.02	−13	7.4	14.7
123	.....	663	.....	.....	8.05	0	8.0	15.3
124	.....	365	.....	193	10.55	+12	2.9	10.2
125	91	120	.....	63	9.28	+24	7.7	15.0
126	.....	52	.....	.....	(7.4)‡	+35	.....	.....
127	41	50	33	41	9.31	+42	8.0	8.4
128	.....	41	.....	40	9.38	+47	8.1	8.4
129	32	39	.....	.....	9.30	+47	8.1	8.4
130	.....	54	.....	50	9.04	+43	8.1	8.4
131	64	82	.....	39	8.86	+34	8.2	15.5
132	.....	112	.....	61	8.10	+24	8.3	15.6
133	.....	381	.....	.....	10.26	+12	5.1	12.4
134	.....	457	.....	285	7.79	0	8.3	15.6
135	905	1277	.....	749	9.42	−13	7.4	14.7
136	155	204	94	124	8.62	−26	8.1	15.4
137	.....	60	.....	71	9.06	−39	8.0	15.3
138	.....	.....	31	38	9.95	−53	7.8	8.1
139	.....	20	.....	19	9.99	−66	7.9	8.3

central stars and the galactic latitudes of the areas appear in the two columns immediately preceding. In a few cases the central stars are doubles; for these the combined magnitude of the components, counted as a single star, has been used. In two or three instances the central star is too bright for measurement, and has not been counted.

As an illustration, the calculation of the corrections for S.A. 1 follows: The magnitude of the central star falls in the interval 7.0–7.5; the latitude is 28°. From Table XVII,[1] $\log N_{7.5} = 9.484$, $\log N_{7.0} = 9.261$. The average number of stars per square degree in the interval is therefore 0.123. The reciprocals of the areas, in square degrees, of the Mount Wilson and Groningen fields are 8.666 and 16. Reduced to unit area, the counts will therefore be in excess by 8.5 and 15.9 stars, respectively. These numbers accordingly appear in Table V as corrections.

Collection of data for 28 latitude groups.—The derivation of Table VII will now be understood from the following details relating to the first latitude group, which, as shown by Table II, includes Areas 24, 40, 123, and 134. The sums of the counts for these areas, formed from the data in Tables I*a* and I*b* and the completed counts in Table V, are given in the first two lines of Table VI.[2] Below $m = 13.0$ the sums refer to half-magnitude intervals for which the column heading is the lower limit. The cumulative sums are in the third and fourth lines, and their logarithms, reduced to an area of 1 square degree, in the fifth and sixth lines. The constants for reduction of the logarithms to unit area are

$$\log (8.666/4) = 0.336\ (MW); \quad \log (16/4) = 0.602\ (Gr).$$

The corrections for the central stars are most easily applied with the aid of subtraction logarithms. The total excess for the four areas, from the last two columns of Table V, is 25.7 for Mount Wilson and 55.0 for Groningen. The logarithms of the corrections are, therefore,

$$\log (25.7/4) = 0.808\ (MW) \text{ and } \log (55.0/4) = 1.138\ (Gr).$$

Combined with the fifth and sixth lines, these give the corrected values of $\log N_m$, which are the final results from the Mount Wilson and Groningen counts. The means of the corrected logarithms[3]

[1] For convenience, this table is used for the illustration instead of van Rhijn's.

[2] In combining the Groningen counts for the group at 40° an obvious modification is necessary to take account of the change in the area of the fields at latitude 40°.

[3] Strictly speaking, the logarithms of the mean of the numbers should be used; but the numbers are so nearly alike that the difference, even when appreciable, is unimportant. The distinction is that between the arithmetical and geometrical means.

appear in the first line of Table VII, and their differences $MW-Gr$ in the first line of Table VIII.

The counts for one area (No. 110) have been omitted altogether. The region is in Serpens, between the two branches of the Milky Way, and so heavily obscured that its inclusion would have affected the mean density seriously.

TABLE VI

DETAILS FOR FIRST LATITUDE GROUP TABLES VII AND VIII

		13.0	13.5	14.0	14.5	15.0	15.5
Counts	M. W.	99	43	54	102	151	240
	Gr.	63	26	34	58	79	127
Cum. sums	M. W.	99	142	196	298	449	689
	Gr.	63	89	123	181	260	387
Log N_m	M. W.	2.332	2.488	2.628	2.810	2.988	3.174
	Gr.	2.401	2.551	2.692	2.860	3.017	3.190
Corr. log N_m	M. W.	2.319	2.479	2.621	2.806	2.985	3.172
	Gr.	2.377	2.534	2.680	2.852	3.011	3.186
Mean table VII		2.348	2.506	2.650	2.829	2.998	3.179
Diff. table VIII		−0.058	−0.055	−0.059	−0.046	−0.026	−0.014

		16.0	16.5	17.0	17.5	18.0	18.5
Counts	M. W.	315	550	674	813	1110	1700
	Gr.	189	296	410	526	702	976
Cum. sums	M. W.	1004	1554	2228	3041	4151	5851
	Gr	576	872	1282	1808	2510	3486
Log N_m	M. W.	3.338	3.527	3.684	3.819	3.954	4.103
	Gr.	3.362	3.543	3.710	3.859	4.002	4.144
Corr. log N_m	M. W.	3.337	3.526	3.683	3.819	3.954	4.103
	Gr.	3.359	3.541	3.709	3.858	4.001	4.144
Mean table VII		3.348	3.534	3.696	3.838	3.978	4.124
Diff. table VIII		−0.022	−0.015	−0.026	−0.039	−0.047	−0.041

The differences in Table VIII afford a useful check. If the fields and the observed magnitudes for the two series of counts were the same, these differences would be zero. Actually, neither of the conditions specified is wholly satisfied, and appreciable systematic differences are to be expected, especially for small groups of four or five areas; but, as a whole, the two series should agree closely, as in fact they do, except for a small but persistent systematic difference. The mean differences for latitudes 0°–20°, 24°–42°, 46°–79°,

TABLE VII

MEAN OBSERVED LOG Nm

Lat.	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5	18.0	18.5
0°	2.348	2.506	2.650	2.829	2.998	3.179	3.348	3.534	3.696	3.838	3.978	4.124
1	2.269	2.460	2.626	2.805	2.978	3.128	3.306	3.493	3.664	3.856	4.019	4.171
3	2.016	2.234	2.408	2.590	2.756	2.916	3.086	3.268	3.462	3.628	3.814	3.978
8	2.034	2.172	2.388	2.610	2.825	3.036	3.222	3.430	3.632	3.772	3.930	4.076
11	1.917	2.118	2.348	2.532	2.747	2.964	3.162	3.325	3.542	3.716	3.866	4.015
12	1.882	2.085	2.372	2.556	2.773	2.949	3.141	3.352	3.528	3.692	3.858	4.013
13	1.951	2.179	2.324	2.564	2.778	2.937	3.136	3.354	3.524	3.738	3.948	4.114
15	1.855	2.172	2.433	2.628	2.790	2.972	3.125	3.302	3.470	3.622	3.767	3.922
18	2.011	2.258	2.404	2.517	2.674	2.854	3.006	3.154	3.335	3.447	3.584	3.730
20	1.681	1.909	2.074	2.251	2.510	2.700	2.850	2.994	3.160	3.295	3.440	3.576
24	1.924	2.114	2.217	2.334	2.512	2.656	2.808	3.024	3.180	3.310	3.500	3.636
26	1.712	1.933	2.138	2.326	2.520	2.716	2.869	3.012	3.164	3.323	3.459	3.590
27	1.726	1.938	2.038	2.234	2.404	2.550	2.720	2.840	3.030	3.178	3.300	3.417
29	1.838	2.008	2.147	2.336	2.484	2.672	2.822	2.988	3.119	3.256	3.384	3.500
33	1.784	1.932	2.092	2.278	2.422	2.566	2.710	2.854	2.962	3.080	3.201	3.314
35	1.763	1.930	1.986	2.134	2.288	2.468	2.614	2.751	2.895	3.029	3.162	3.282
39	1.552	1.830	1.950	2.086	2.312	2.439	2.555	2.663	2.804	2.882	3.012	3.139
40	1.654	1.816	1.996	2.166	2.308	2.442	2.598	2.744	2.909	3.036	3.158	3.274
42	1.594	1.788	1.902	2.042	2.178	2.353	2.470	2.607	2.756	2.930	3.062	3.194
46	1.460	1.707	1.899	2.042	2.189	2.359	2.474	2.586	2.734	2.854	2.996	3.121
48	1.663	1.850	1.972	2.108	2.270	2.392	2.519	2.650	2.778	2.931	3.042	3.152
51	1.574	1.698	1.851	1.973	2.187	2.314	2.468	2.606	2.724	2.830	2.980	3.090
54	1.438	1.710	1.928	2.022	2.179	2.295	2.410	2.522	2.665	2.800	2.927	3.032
59	1.662	1.766	1.870	2.032	2.153	2.288	2.388	2.512	2.614	2.726	2.876	2.988
61	1.538	1.628	1.798	1.966	2.134	2.268	2.382	2.504	2.634	2.736	2.859	2.978
66	1.432	1.596	1.692	1.836	1.962	2.109	2.250	2.408	2.521	2.652	2.835	2.946
73	1.486	1.668	1.782	1.932	2.052	2.138	2.231	2.388	2.518	2.632	2.772	2.883
79	1.248	1.514	1.732	1.876	2.050	2.144	2.262	2.360	2.485	2.591	2.698	2.820

TABLE VII*a*

DEVIATION OF TABLE VII FROM ADOPTED MEAN DISTRIBUTION OF TABLE XVII

Lat.	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5	18.0	18.5
0°	+0.185	+0.138	+0.081	+0.063	+0.039	+0.032	+0.018	+0.027	+0.017	−0.008	−0.030	−0.041
1	+ .115	+ .102	+ .067	+ .049	+ .029	− .010	− .015	− .006	− .007	+ .018	+ .019	+ .014
3	− .119	− .105	− .131	− .146	− .174	− .203	− .217	− .214	− .193	− .195	− .171	− .163
8	− .033	− .097	− .079	− .053	− .030	− .008	− .007	+ .022	+ .050	+ .022	+ .018	+ .008
11	− .100	− .098	− .065	− .075	− .050	− .020	− .005	− .019	+ .025	+ .032	+ .021	+ .016
12	− .117	− .113	− .022	− .031	− .003	− .012	− .002	+ .033	+ .038	+ .036	+ .043	+ .044
13	− .031	.000	− .050	− .002	+ .024	− .002	+ .018	+ .061	+ .061	+ .111	+ .162	+ .176
15	− .091	+ .030	+ .098	+ .103	+ .079	+ .079	+ .055	+ .060	+ .061	+ .051	+ .040	+ .045
18	+ .112	+ .166	+ .123	+ .051	+ .028	+ .032	+ .013	− .005	+ .015	− .029	− .042	− .041
20	− .187	− .150	− .171	− .175	− .092	− .074	− .091	− .109	− .100	− .117	− .119	− .125
24	+ .111	+ .113	+ .034	− .025	− .018	− .040	− .050	+ .011	+ .017	+ .003	+ .055	+ .058
26	− .074	− .040	− .015	− .001	+ .023	+ .055	+ .050	+ .040	+ .046	+ .064	+ .066	+ .068
27	− .048	− .021	− .101	− .079	− .077	− .094	− .082	− .113	− .068	− .059	− .069	− .079
29	+ .089	+ .076	+ .036	+ .052	+ .034	+ .061	+ .055	+ .073	+ .062	+ .064	+ .062	+ .055
33	+ .083	+ .050	+ .033	+ .049	+ .029	+ .015	+ .008	+ .008	− .022	− .036	− .041	− .047
35	+ .085	+ .072	− .047	− .068	− .077	− .053	− .056	− .062	− .055	− .051	− .042	− .039
39	− .082	+ .018	− .035	− .066	.000	− .027	− .057	− .090	− .083	− .132	− .123	− .110
40	+ .031	+ .015	+ .023	+ .027	+ .009	− .010	.000	+ .006	+ .038	+ .038	+ .040	+ .043
42	− .012	+ .005	− .052	− .076	− .099	− .076	− .103	− .105	− .088	− .040	− .027	− .008
46	− .112	− .039	− .016	− .035	− .044	− .023	− .050	− .075	− .057	− .061	− .036	− .022
48	+ .108	+ .122	+ .077	+ .051	+ .059	+ .034	+ .019	+ .015	+ .014	+ .044	+ .038	+ .038
51	+ .044	− .003	− .016	− .053	+ .008	− .011	+ .003	+ .007	− .002	− .018	+ .016	+ .016
54	− .067	+ .035	+ .088	+ .024	+ .029	.000	− .024	− .045	− .030	− .016	− .004	− .008
59	+ .198	+ .134	+ .076	+ .082	+ .052	+ .043	+ .005	− .003	− .028	− .036	.000	+ .004
61	+ .088	+ .012	+ .020	+ .032	+ .051	+ .041	+ .018	+ .008	+ .012	− .006	+ .004	+ .015
66	+ .013	+ .013	− .051	− .061	− .082	− .077	− .071	− .043	− .054	− .042	+ .028	+ .031
73	+ .105	+ .124	+ .081	+ .079	+ .054	+ .001	− .040	− .010	− .003	− .007	+ .021	+ .025
79	−0.108	−0.003	+0.058	+0.052	+0.082	+0.038	+0.024	−0.006	−0.003	−0.014	−0.018	−0.001

TABLE VIII

DIFFERENCES, $MW-Gr$, IN OBSERVED LOG N_m

Lat.	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5	18.0	18.5
0°	−0.058	−0.055	−0.059	−0.046	−0.026	−0.014	−0.022	−0.015	−0.026	−0.039	−0.047	−0.041
1	− .004	− .023	− .016	− .034	− .029	− .033	− .033	− .016	− .018	− .004	.000	+ .006
3	+ .043	+ .024	+ .030	+ .032	+ .023	+ .019	+ .016	+ .015	+ .021	+ .013	+ .014	.000
8	+ .061	+ .118	+ .094	+ .061	+ .048	+ .029	+ .001	+ .009	+ .013	+ .003	− .012	+ .003
11	− .078	− .025	− .021	− .040	− .008	+ .003	− .035	− .014	+ .017	+ .024	− .029	− .060
12	+ .119	+ .052	− .002	− .003	− .002	− .010	− .010	− .009	− .012	− .002	− .007	− .010
13	− .070	− .036	− .030	− .028	− .022	− .006	− .003	+ .002	− .021	− .056	− .100	− .088
15	+ .154	+ .035	.000	− .019	− .019	− .027	− .008	+ .008	+ .024	− .006	.000	+ .013
18	+ .042	− .028	− .014	+ .002	+ .001	+ .011	+ .012	+ .014	+ .008	− .006	+ .004	+ .011
20	+ .018	− .010	+ .058	+ .100	+ .012	− .005	+ .020	+ .029	+ .043	+ .048	+ .032	+ .043
24	− .119	− .071	− .044	− .022	− .005	− .010	− .030	+ .009	− .022	+ .005	− .015	− .007
26	+ .048	+ .052	+ .036	− .026	− .015	− .015	− .028	− .025	.000	− .024	− .008	− .001
27	− .088	− .040	− .032	− .028	− .002	− .003	− .001	− .006	− .017	− .027	− .004	+ .010
29	− .039	− .043	− .032	− .035	− .029	− .033	− .019	− .024	− .006	− .025	− .018	− .006
33	+ .034	− .016	− .011	− .083	− .039	− .038	− .041	− .017	+ .021	− .011	+ .022	+ .049
35	− .026	− .066	− .038	− .059	− .069	− .016	− .002	− .008	+ .004	+ .004	− .010	+ .005
39	+ .155	+ .034	+ .019	+ .017	+ .009	+ .006	+ .014	+ .016	+ .020	+ .025	+ .002	+ .030
40	− .054	+ .016	+ .005	− .003	− .013	− .016	− .036	− .037	− .002	+ .004	+ .013	+ .019
42	− .054	− .028	− .035	− .029	− .004	− .006	− .003	+ .002	− .003	− .009	− .005	+ .015
46	− .015	− .002	− .006	− .016	− .016	− .016	− .021	− .024	− .015	− .013	.000	+ .006
48	− .032	− .005	+ .009	+ .021	+ .017	+ .009	− .004	.000	− .001	− .036	− .038	− .040
51	− .035	− .031	− .026	− .008	+ .028	+ .028	+ .031	+ .037	+ .023	+ .022	− .025	− .015
54	+ .039	+ .028	− .007	− .016	− .016	.000	− .010	− .003	− .012	− .037	− .058	− .061
59	− .066	− .029	− .047	− .037	− .026	− .036	− .010	− .007	− .007	− .004	− .031	− .020
61	+ .029	+ .003	+ .033	+ .035	+ .001	+ .004	− .016	− .026	− .004	− .028	− .046	− .069
66	− .072	− .056	− .032	− .004	− .025	− .046	− .003	+ .017	− .004	− .029	− .042	− .045
73	+ .035	+ .017	+ .035	+ .019	+ .031	+ .012	+ .002	+ .009	+ .007	− .039	− .030	− .034
79	+0.027	+0.032	−0.015	−0.036	−0.057	−0.021	−0.057	−0.030	−0.028	−0.014	−0.019	−0.029
0°–20°	+0.023	+0.005	+0.004	+0.002	−0.002	−0.003	−0.006	+0.002	+0.005	−0.002	−0.014	−0.012
24 –42	− .016	− .018	− .015	− .030	− .019	− .015	− .016	− .010	− .001	− .006	− .003	+ .013
46 –79	− .010	− .005	− .006	− .005	− .007	− .007	− .010	− .003	− .005	− .020	− .032	− .034
0 –79	0.000	−0.005	−0.005	−0.010	−0.009	−0.008	−0.011	−0.003	0.000	−0.009	−0.016	−0.011

and 0°–79°, given in the last four lines of Table VIII, show no progression either in latitude or in magnitude. The mean, Mount Wilson *minus* Groningen, for all latitudes and magnitudes is −0.007. Interpreted as a difference in zero-points, this corresponds to 0.02 mag.; as an error in the areas of the fields, it corresponds to a systematic difference of about 10″ in the diameters of the fields of one series relative to those of the other.

Provisional mean distribution from the "Mount Wilson Catalogue." —The combined Mount Wilson and Groningen counts in Table VII, although mean values for five widely separated regions, are still much affected by local irregularities of distribution, which, in the case of small fields, are frequently a large percentage of the total star density. A further step toward the mean distribution was made by plotting the data in each column of Table VII against the corresponding latitude. The resulting curves represent the variation in log N_m with galactic latitude for each half-magnitude value of m from 13.0 to 18.5. These are reproduced in Figure 1, which shows the character and general order of the irregularities in the data of Table VII. The deviations for any given group of areas are naturally much alike for all the curves. A conspicuous case is the group for 3°. Special examination shows that all the fields are more or less obscured, which accounts for the persistently low density evident for all magnitudes. The fluctuations from group to group for any given magnitude are, however, accidental, and of such a character that the variation in log N_m with latitude is well defined. Thus, inspection of Figure 1 shows that a separate plotting of the data for alternate latitude groups in Table VII would have led to curves practically identical with those found from all the groups. The irregularities affecting the individual groups are still better shown by Table VII*a*, which gives the deviations from the adopted mean distribution in Table XVII.

To eliminate errors in the spacing of the curves of Figure 1 their ordinates were tabulated with values of m as argument for each 10° of latitude and adjusted until the first differences became regular. The smoothed ordinates for the interval $m=13.5$ to 18.5 are in Table IX, which represents the provisional mean distribution derived from the *Mount Wilson Catalogue.*

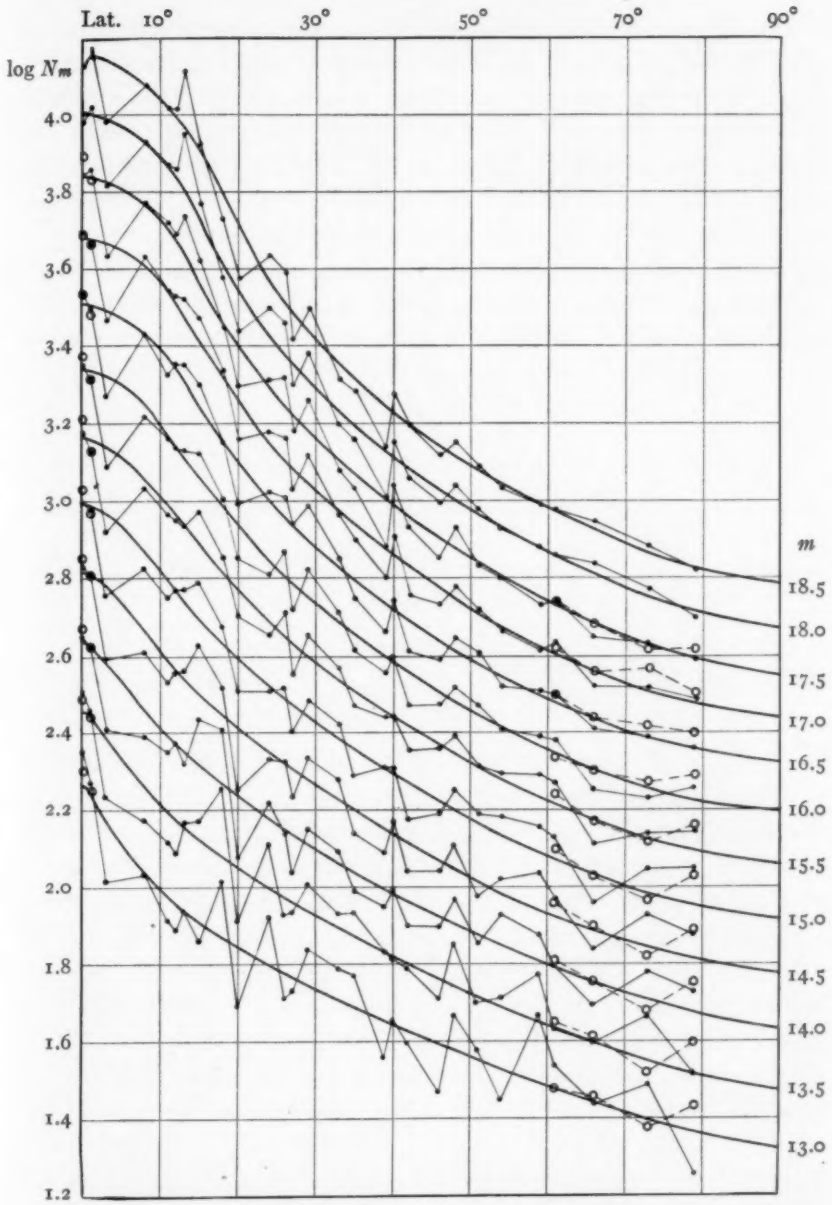


FIG. 1.—Observed values of log N_m for different galactic latitudes and magnitudes. Points are from Table VII, based on the *Mount Wilson Catalogue of Selected Areas;* circles represent counts from the *Harvard-Groningen Durchmusterung.* The curves are the basis of Table IX.

Local irregularities have now been rather thoroughly eliminated. Any residual effect will, however, be systematic, and is most likely to appear in the brighter magnitudes and in low galactic latitudes. The number of stars counted to the brighter limits of magnitude is small, even in the Milky Way; and in low latitudes the distribution in galactic longitude is not representative. Thus for latitudes less than 13° no counts whatever are available for stars fainter than $m = 13$ between longitudes 190° and 350°. This is an unavoidable defect in the data. The order of the uncertainty will appear later when the results are combined with those from the astrographic zones.

TABLE IX

SMOOTHED VALUES OF LOG N_m FROM TABLE VII

m	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
13.5	2.445	2.210	2.047	1.924	1.817	1.716	1.632	1.562	1.504	1.469
14.0	2.629	2.414	2.233	2.097	1.981	1.874	1.786	1.713	1.656	1.624
14.5	2.811	2.615	2.415	2.266	2.140	2.026	1.935	1.861	1.805	1.774
15.0	2.990	2.813	2.594	2.429	2.295	2.174	2.080	2.003	1.948	1.919
15.5	3.166	3.007	2.768	2.586	2.444	2.318	2.220	2.142	2.087	2.058
16.0	3.340	3.197	2.937	2.740	2.587	2.458	2.356	2.275	2.220	2.193
16.5	3.510	3.381	3.101	2.887	2.727	2.593	2.488	2.405	2.349	2.322
17.0	3.677	3.557	3.260	3.027	2.861	2.724	2.616	2.531	2.474	2.447
17.5	3.841	3.727	3.413	3.161	2.990	2.852	2.741	2.655	2.594	2.567
18.0	4.001	3.890	3.559	3.289	3.114	2.975	2.863	2.775	2.710	2.682
18.5	4.156	4.045	3.699	3.410	3.232	3.093	2.980	2.891	2.822	2.792

Comparison with the "Harvard-Groningen Durchmusterung."—The *Harvard-Groningen Durchmusterung* of the Selected Areas gives the magnitudes of about a quarter of a million stars brighter than the sixteenth magnitude in the 206 Selected Areas between the North and South poles. The fields are 40′×40′, 60′×60′, and 80′×80′, according to the galactic latitude, and hence, on the average, some ten times the area of the fields of the *Mount Wilson Catalogue*. When the scale corrections for the areas south of declination −15° are better known, the *Durchmusterung* will be especially useful in deriving the distribution in the southern sky. Scale corrections for the areas north of −15° are already available,[1] and this part of the *Durchmusterung* might have been utilized in the present discussion. In fact, it has been used to some extent in extrapolating the curves of Figure 1 to 90°, but chiefly in a comparison of results for the 29

[1] Seares, Joyner, and Richmond, *Mt. Wilson Contr.*, No. 289; *Astrophysical Journal*, **61**, 303, 1925.

areas in the first two and the last four latitude groups of Table VII. The relatively large fields of the *Durchmusterung* undoubtedly would have smoothed out some of the irregularities appearing in Figure 1; but the comparison shows that the mean distribution defined by the curves themselves would have been little affected. Figure 1, in which circles indicate data from the *Durchmusterung*, also shows the agreement.

The stars in the 29 areas were counted for half-magnitude intervals in the usual manner and summed to $m = 15.5$ or 16.0, according to the limiting magnitude. Curves derived by plotting the logarithms of the cumulative sums against magnitude were then

TABLE X

COMPARISON OF LOG N_m FOR *Mount Wilson Catalogue* AND *Harvard-Groningen Durchmusterung*. (*Cat.*−*DM.*)

Group	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5
0°.....	+0.05	+0.02	−0.02	−0.02	−0.03	−0.03	−0.02	0.00	0.00	−0.05
1.....	+ .02	+ .02	.00	+ .01	+ .01	− .02	− .01	+ .01	+ .01	+ .03
61.....	+ .06	− .02	− .01	+ .01	+ .03	+ .03	+ .01	.00	+ .01	.00
66.....	− .02	− .02	− .06	− .06	− .07	− .06	− .06	− .03	− .04	− .02
73.....	+ .11	+ .15	+ .11	+ .11	+ .08	+ .02	− .04	− .03	− .05	− .09
79.....	−0.18	−0.08	−0.02	−0.01	+0.02	−0.02	−0.02	−0.04	−0.02	−0.03
Mean.....	+0.01	+0.01	0.00	+0.01	+0.01	−0.01	−0.02	−0.02	−0.02	−0.03

corrected graphically for scale error by means of Table III, *Contribution* No. 289, and extrapolated to $m = 17.5$. The ordinates read from these curves, reduced to an area of 1 square degree, gave values of N_m for each area which were then grouped into means according to Table II. Finally, the logarithms of the mean values of N_m were compared with the corresponding values of log N_m in the first two and the last four lines of Table VII, with the results shown in Table X. In view of the difference in the areas counted, the agreement is satisfactory. Even the large systematic differences for the group at 73° represent irregularities of only a few stars in the counts from the *Mount Wilson Catalogue*. The mean differences for the six groups are small, and show that the adopted distribution would have been little affected by including counts from the *Durchmusterung*.

An important result of the comparison is that the differences in the last three columns of Table X, all of which depend on an extrapolation of the *Durchmusterung* counts, are comparable with those

in the other columns. The possibility of such an extrapolation is used later to extend the mean distribution table to the twenty-first magnitude.

3. CORRECTION OF VAN RHIJN'S DISTRIBUTION TABLE FOR SCALE ERROR

Table IV of *Groningen Publication* No. 27 plays an important part in the determination of the distribution of stars brighter than $m = 13.5$. For the stars fainter than $m = 10$ this table is based on 65 northern Selected Areas of the *Harvard-Groningen Durchmusterung*.

TABLE XI

REDUCTION OF TABLES II AND IV, *Groningen Publication* No. 27, TO INTERNATIONAL SCALE

Gr. 27 Mag.	Scale Corr.	Corr. Mag.	*Gr.* 27 Mag.	Scale Corr.	Corr. Mag.
4.0	−0.08	3.92	10.5	+0.39	10.89
4.5	− .08	4.42	11.0	.40	11.40
5.0	− .07	4.93	11.5	.40	11.90
5.5	− .05	5.45	12.0	.40	12.40
6.0	.00	6.00	12.5	.40	12.90
6.5	+ .05	6.55	13.0	.38	13.38
7.0	+ .10	7.10	13.5	.36	13.86
7.5	+ .15	7.65	14.0	.33	14.33
8.0	+ .20	8.20	14.5	.30	14.80
8.5	+ .25	8.75	15.0	.27	15.27
9.0	+ .29	9.29	15.5	.22	15.72
9.5	+ .33	9.83	16.0	+0.16	16.16
10.0	+0.38	10.38			

For stars brighter than $m = 5.5$ it depends on counts from *Harvard Annals*, **50**, reduced to the photographic scale by correction for color-index. The *Göttingen Aktinometrie* was used to cover the interval $m = 5.5$ to 7.0, and Greenwich astrographic magnitudes from $m = 6.5$ to 8.5 (latitudes 10° to 50° only). The remaining interval in the scale of brightness was bridged by interpolation.

Below $m = 10$ the scale correction to the magnitudes of van Rhijn's table is that given in the first column of Table IV, *Contribution* No. 289, while for $m < 6$ it is that of the Harvard visual scale, given in Table II, *Contribution* No. 288,[1] modified by a small color equation which may be neglected here. The Greenwich astrographic

[1] *Astrophysical Journal*, **61**, 284, 1925.

magnitudes are on the Harvard photographic scale, for which the reduction to the international system, in the region in question, is similar to that for the Harvard visual scale. The *Aktinometrie* has little influence on the scale error of van Rhijn's table, and may be disregarded.

For practical purposes we may adopt the corrections specified for $m<6$ and >10, and cover the interval $m=6$ to 10 by interpolation. The results, smoothed by 0.01 or 0.02 mag., are in the second column of Table XI. The corrected arguments for van Rhijn's table are in the last column of this table.

TABLE XII

TABLE XVII MINUS *Groningen Publication* NO. 27, TABLE IV, CORRECTED FOR SCALE

m	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
9	+0.03	+0.02	−0.01	−0.01	−0.02	+0.01	+0.01	+0.01	+0.01	0.00
10	.06	.03	.01	− .03	− .03	.00	.01	.01	+ .01	.00
11	.06	.02	.03	− .03	− .03	.00	.00	.00	− .01	.00
12	.07	.00	.05	− .03	− .03	.01	.00	.00	− .01	.00
13	.09	.00	.05	− .02	− .01	.04	.03	.00	− .02	− .03
14	.13	.02	.03	+ .01	+ .04	.08	.03	.02	.00	.00
15	.15	.05	.01	+ .04	+ .06	.07	.05	.02	.00	.00
16	+0.17	+0.07	−0.03	+0.02	+0.04	+0.05	+0.02	+0.01	−0.01	−0.01

Van Rhijn's table thus corrected was compared with the provisional mean distribution for the interval $m=13.5$–18.5, derived from the *Mount Wilson Catalogue* and given in Table IX. No serious systematic differences were found outside the Milky Way, from which it was inferred that the corrected table must also be near the truth for the interval $m=9.0$ to 13.5, covered by the astrographic zones. This inference was afterward justified by a comparison with the adopted distribution given in Table XVII, the details of which, in Table XII, are inserted at this point.

4. COUNTS FROM THE "ASTROGRAPHIC CATALOGUE"

Ten zones of the *Astrographic Catalogue*[1] were used in 1917 to determine the galactic concentration in the interval $m=8.5$ to 12.5. The values of $\log N_m$ were necessarily referred to the magnitude scale used by Kapteyn, which even then was known to be seriously in error. Although the results confirmed the rapid increase in concentration with magnitude found by Kapteyn in *Groningen Publication*

[1] Seares, *Mt. Wilson Contr.*, No. 135; *Astrophysical Journal*, **46**, 117, 1917.

No. 18, the absolute values of the density were approximate. With a reliable scale available, the method used in *Contribution* No. 135 can now be applied very effectively to the abundant material from the *Astrographic Catalogue* for a determination of the mean distribution of stars in the interval $m = 9.0$ to 13.5.

Thanks largely to Professor Turner, counts for 33 zones including about 1,400,000 stars[1] are now available. The declinations of the zones, the observatories concerned, and references to the volumes of the *Monthly Notices* giving the results are collected in Table XIII. The counts are referred to the arbitrary scales of magnitude or diameter of images used in preparing the zones for publication. In using the material it is necessary to assume that a given scale reading[2] always corresponds to the same magnitude on the international scale. For various reasons this is by no means always the case. Differences in exposure time; in observing conditions, both accidental and seasonal; in the sensitiveness of the plates; and variations in the measures themselves arising from subjective factors all tend to invalidate the assumption; but the material is now so extensive that in the mean these factors operate largely as accidental disturbances.

The published counts, usually the numbers of stars in each hour of right ascension in successive intervals of scale reading, have been treated just as in *Contribution* No. 135, except that the exact area of an astrographic plate, instead of the approximation of 4 square degrees, has been used. Some of the original curves have been revised, and data for additional limiting magnitudes have been introduced in a few cases. Logarithms of the cumulative sums for an area of 1 square degree were plotted against galactic latitude for four or five limiting values of the scale reading, and smoothed values of log N_m for latitudes 0°, 10°, 20°, ... were read from the resulting curves. Known regions of obscuration, such as that in Taurus, were disregarded in drawing the curves.

[1] The totals actually counted in the zones used, as listed in Table XIII, sum up to about 1,810,000. Allowance for overlapping zones reduces the number to that mentioned.

[2] This term is used to designate the values of the brightness of the stars as given in the zones, irrespective of whether they represent readings with a comparison scale, diameters of images, or estimates of magnitude.

TABLE XIII

DATA FOR ASTROGRAPHIC ZONES

Zone	Observatory	Reference *Monthly Notices*	No. Stars Counted	No. Plates per Hour
+64°	Vatican			
+62	Vatican	75,601	88,782	10
+60	Vatican			
+31	Oxford	85,471	69,332	7,6,7
+30	Oxford	85,471	83,747	7,7,6
+29	Oxford	85,471	68,062	7,6,7
+28	Oxford	75,465	70,141	7,7,6
+27	Oxford	85,471	62,411	7,8
+26	Oxford	85,471	73,764	8,7
+25	Oxford	85,471	76,402	7,8
+24	Paris	83,385	.....	.....
+23	Paris	83,385	70,538	7,8
+22	Paris	83,385	72,285	8,7
+17	Bordeaux	72,464	49,747	7,8
+16	Bordeaux	79,136	40,183	8,7
+15	Bordeaux	79,136	36,746	7,8
+14	Bordeaux	79,136	53,784	8,7
+ 9	Toulouse	76,149	35,700	See note
− 1	Algiers	72,700	41,322	7,8
− 3	San Fernando	83,385	56,117	7,8
− 4	San Fernando	84,735	41,441	8,7
− 5	San Fernando	83,385	46,279	7,8
− 6	San Fernando	84,735	39,466	8,7
−15	Tacubaya	78,54	63,945	7,8
−16	Tacubaya	79,565	63,106	8,7
−17	Hyderabad	77,432	63,000	See note
−18	Hyderabad	78,367	61,379	See note
−19	Hyderabad	80,623	58,500	See note
−25	Cordoba	78,54	68,764	7,8
−27	Cordoba	81,525	54,584	7,8
−29	Cordoba	81,525	49,552	7,6,7
−31	Cordoba	81,525	61,162	7,6,7
−32	Perth	75,143	59,629	See note
−34	Perth	78,578	63,050	See note
−36	Perth	80,620	68,535	See note
−38	Perth	78,578	68,453	6
−41	Cape	75,57	71,433	6
−42	Cape	79,565	84,217	6
−43	Cape	79,565	73,781	6
−65	Melbourne	77,35	33,813	3,4,3

NOTES TO TABLE XIII

+62°: The counts are means for zones +60°, +62°, and +64°, extending from +59° to +65°. Unit=0.1 star per plate. Since in the three zones together there are 10 plates per hour, the counts are also numbers of stars per hour.

+28°: Counts for +28° only of the Oxford zones were available when the reductions were made. For completeness, references are given for the other zones, which have recently appeared. The abnormally low densities in the obscured region, 3^h–5^h, +28°, have been disregarded in deriving the means given in Table XV.

NOTES TO TABLE XIII—*Continued*

+24°: Detailed counts have not been published.

+23°, +22°: Obscured region (see +28°) disregarded. For 6, 8, 18, and 22^h the published values under the heading "All" do not agree with the sums of the numbers in the successive intervals of scale reading. The individual numbers are assumed to be correct.

+17° to +14°: All the Bordeaux zones show abnormally low densities in the region of 4^h.

+9°: The scale for 0^h–6^h differs from that for 7^h–23^h, the densities for the first quadrant being relatively low. See comment by Turner, *loc. cit.* Total number of plates is 180; an average of 7.5 per hour has been used for the reduction.

−1°: The sign of the declination, wrongly given with the original counts, was later corrected (cf. *M.N.*, **84**, 738). The densities from 12^h to 18^h, galactic latitudes +60° to +3°, are nearly constant. Since the San Fernando zones, −3° to −6°, show a similar phenomenon, the abnormality is probably real.

−3° to −6°: See preceding note.

−15°: Density from 12^h to 17^h, galactic latitude +46° to +8°, nearly constant. The published total, *loc. cit.*, for scale reading 10.5 should be 13377. The values of N' are correspondingly affected.

−16°: Density from 0^h to 4^h, galactic latitude −78° to −36°, nearly constant.

−17°: Data published by Pocock. The number of plates is 8 and 7 per hour, but the counts have been reduced to a total of 10 per hour. The number of stars given is approximate.

−18°: Data published by Pocock. Cumulative sums, reduced to a total of 10 plates per hour, calculated from Table I of Pocock's paper.

−19°: Data published by Bhaskaran. Number of plates, 7 and 8; counts reduced to 10 plates per hour. Number of stars approximate.

−32° to −36°: Number of plates, 6 and 7; counts reduced to 10 per hour. Totals counted approximate.

−41° to −43°: The density for all three zones at 12^h, galactic latitude +20°, is conspicuously low as compared with that at 18^h, latitude −16°.

−65°: The systematic difference in scale readings discussed by Turner, *loc. cit.*, affects all hours of right ascension, and is therefore probably well eliminated from the mean densities in Table XV.

As an illustration, the mean values of log N_m thus found for the Vatican zones, +62°, are given in Table XIV under the scale readings 40, 30, ... 0. Each of these, in conjunction with van Rhijn's Table IV, corrected as in the preceding section, determines a value of the limiting magnitude corresponding to the scale reading at the head of the column. The differences in Table XII show, however, that magnitudes interpolated for latitude 0° cannot safely be used. These were accordingly rejected in forming the mean limiting magnitudes. As a matter of convenience, the graphical equivalent of van Rhijn's table given in Figure 2, *Groningen Publication* No. 27,

was used for the interpolation, the correction for scale being applied to the mean of the resulting magnitudes. Thus for the Vatican zones the mean magnitude for each scale reading, the scale correction from Table XI, and the corrected magnitude are in the last three lines of Table XIV.

The residuals for the individual values of the magnitude are in the right half of the table. In view of the numerous possible sources of error, these are smaller than might have been expected. Since the part of van Rhijn's table used shows no serious systematic

TABLE XIV

MEAN LOG N_m, LIMITING MAGNITUDE, AND RESIDUALS IN m FOR VATICAN ZONES

GAL. LAT.	SCALE READING					SCALE READING				
	40	30	20	10	0	40	30	20	10	0
0°.....	0.56	0.98	1.40	1.89	2.15	.....	.....	.....	.....	.....
10.....	0.42	.87	1.28	1.77	2.03	−0.34	−0.21	−0.07	−0.01	−0.02
20.....	0.21	.70	1.17	1.67	1.90	− .15	− .04	− .06	− .03	.00
30.....	0.01	.55	1.03	1.53	1.74	+ .08	+ .08	.00	− .06	+ .05
40.....	9.90	.47	0.93	1.39	1.60	+ .13	+ .06	+ .01	.00	+ .02
50.....	9.83	.40	0.83	1.27	1.51	+ .16	+ .04	+ .03	+ .02	− .03
60.....	9.78	0.34	0.75	1.19	1.44	+0.15	+0.05	+0.07	+0.06	−0.03
Mean mag.	8.56	9.69	10.71	11.92	12.57	.....	.....	.....	.....	.....
Sc. corr....	+0.25	+0.35	+0.39	+0.40	+0.40	.....	.....	.....	.....	.....
Int. mag...	8.81	10.04	11.10	12.32	12.97	.....	.....	.....	.....	.....

error (see Table XII), large progressive changes in the residuals, like those for scale reading 40, must originate in the astrographic counts. Whether these represent systematic errors in scale reading or real deviations from the assumed symmetrical mean distribution of the stars cannot be decided at present.

The other zones listed in Table XIII were treated in precisely the same manner. The results for all the zones are collected in Table XV in order of limiting magnitude. Thus opposite magnitudes 8.81, 10.04, etc., in the first column stand the values of log N_m for different galactic latitudes given in the successive columns of Table XIV. To simplify the combination of these data, missing values of the density for high latitudes have been supplied by extrapolation. These are inclosed in parentheses.

It should be noted that the variation of density for any magnitude in Table XV depends solely on the counts from the astrographic zones. Van Rhijn's table has been used only to determine the mean limiting magnitudes, and these are little affected by residual errors in his corrected table. Thus a redetermination of the limits by comparing all the densities in Table XV with the adopted distribution in Table XVII gives a maximum difference of 0.09 and a mean of ±0.027 mag. The revised limiting magnitudes, it may be remarked, provide a mean calibration of the provisional scales of the astrographic zones which may be used until a more detailed reduction to the international scale is available. These results will be collected in a later *Contribution*.

The 128 sequences of log N_m in Table XV were combined into the 11 means given in Table XVI.[1] The division lines in Table XV show the zones contributing to each mean; the place of observation can be identified by means of Table XIII. In general, each mean sequence depends on data from widely separated parts of the sky, based on measures made at several different observatories. Systematic errors affecting the counts of individual zones and deviations from the mean distribution have therefore little influence on Table XVI, which is the final form for the data from the astrographic zones. Residual errors are still further reduced by the smoothing process involved in the combination with data from other sources to form the mean distribution table.

5. THE MEAN DISTRIBUTION TABLE

Tables IX and XVI, based on the *Mount Wilson Catalogue* and the astrographic zones, determine the distribution for stars fainter than $m=9.0$. As already stated, the data to be used for stars brighter than $m=9.0$ are those of van Rhijn, reduced to the international scale. For the interval $m=7.0$ to 9.0, the densities can be taken directly from Table IV, *Groningen Publication* No. 27; for still brighter stars, Table II of van Rhijn's paper must be used. The corresponding magnitudes are in both cases the corrected arguments in the third column of Table XI.

[1] Here again, as a matter of convenience, the geometrical mean has been used for N_m, whereas, strictly speaking, the arithmetical mean is required.

TABLE XV

LOG N_m FROM ASTROGRAPHIC ZONES

m	Zone	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
8.62	−27°	0.28	0.23	0.13	9.95	9.85	9.79	9.74	9.72	9.71	9.70
8.66	+16	0.19	0.12	0.02	9.92	9.89	9.86	9.83	9.80	9.78	(9.77)
8.74	−25	0.26	0.21	0.13	0.03	9.94	9.88	9.83	9.78	9.75	9.73
8.81	+62	0.56	0.42	0.21	0.01	9.90	9.83	9.78	(9.75)	(9.73)	(9.72)
8.84	−34	0.58	0.44	0.25	0.12	9.99	9.89	9.83	9.77	9.72	9.68
8.92	+ 9	0.66	0.53	0.28	0.09	9.95	9.86	9.82	9.78	(9.76)	(9.75)
9.15	−31	0.68	0.53	0.28	0.08	0.01	9.99	9.98	9.98	9.98	9.98
9.20	+23	0.41	0.34	0.25	0.17	0.11	0.08	0.05	0.03	0.00	9.98
9.21	− 1	0.52	0.45	0.34	0.23	0.13	0.04	9.98	(9.95)	(9.93)	(9.92)
9.25	−29	0.66	0.55	0.38	0.27	0.18	0.08	0.00	9.94	9.91	9.89
9.26	+22	0.44	0.37	0.28	0.20	0.14	0.10	0.07	0.05	0.03	0.01
9.32	−38	0.63	0.51	0.33	0.18	0.14	0.11	0.08	0.05	0.04	(0.04)
9.38	−36	0.64	0.49	0.33	0.21	0.16	0.13	0.12	0.11	0.10	(0.10)
9.39	−19	0.44	0.40	0.35	0.28	0.21	0.16	0.13	0.10	0.09	(0.09)
9.49	−42	0.63	0.57	0.49	0.39	0.27	0.17	0.11	0.06	0.04	(0.03)
9.53	−17	0.62	0.55	0.44	0.32	0.25	0.20	0.17	0.14	0.13	(0.12)
9.57	−18	0.58	0.47	0.37	0.33	0.28	0.26	0.23	0.20	0.18	(0.18)
9.57	−43	0.71	0.63	0.52	0.39	0.28	0.20	0.16	0.12	0.09	(0.08)
9.66	+15	0.73	0.62	0.51	0.41	0.33	0.26	0.21	0.16	0.13	(0.12)
9.94	−16	0.68	0.63	0.53	0.47	0.42	0.40	0.39	0.37	0.36	(0.36)
9.95	−25	0.94	0.83	0.68	0.55	0.45	0.38	0.30	0.26	0.23	0.21
9.99	+14	0.89	0.82	0.66	0.53	0.45	0.39	0.34	0.30	0.27	(0.26)
10.00	− 4	0.78	0.73	0.65	0.58	0.49	0.39	0.35	0.33	(0.32)	(0.32)
10.02	− 5	0.77	0.73	0.65	0.58	0.48	0.40	0.37	0.35	(0.34)	(0.33)
10.03	−15	0.80	0.73	0.65	0.55	0.48	0.43	0.40	0.37	(0.36)	(0.36)
10.04	+62	0.98	0.87	0.70	0.55	0.47	0.40	0.34	(0.31)	(0.29)	(0.29)
10.04	− 6	0.82	0.77	0.68	0.55	0.46	0.43	0.40	0.37	(0.36)	(0.35)
10.04	−27	0.97	0.90	0.73	0.53	0.44	0.38	0.33	0.31	0.30	0.29
10.04	−38	0.95	0.80	0.64	0.50	0.45	0.42	0.40	0.38	0.36	(0.36)
10.05	+28	0.83	0.78	0.67	0.54	0.47	0.42	0.39	0.38	(0.38)	(0.38)
10.06	+16	0.85	0.77	0.64	0.53	0.48	0.44	0.40	0.37	0.34	(0.33)
10.12	−34	1.08	0.93	0.77	0.62	0.51	0.42	0.37	0.33	0.29	0.25
10.16	− 3	0.88	0.83	0.74	0.63	0.55	0.46	0.41	0.37	(0.36)	(0.35)
10.19	−19	0.87	0.80	0.71	0.61	0.54	0.49	0.45	0.43	0.42	(0.42)
10.28	+ 9	1.20	1.07	0.82	0.65	0.52	0.46	0.42	0.39	(0.37)	(0.36)
10.35	−32	1.08	0.98	0.83	0.67	0.53	0.48	0.47	0.47	0.47	(0.47)
10.38	+22	0.96	0.88	0.76	0.68	0.59	0.55	0.53	0.51	0.47	0.44
10.45	+23	0.93	0.86	0.78	0.68	0.64	0.61	0.57	0.53	0.50	0.48
10.58	−31	1.29	1.07	0.87	0.73	0.64	0.58	0.57	0.55	0.54	0.53
10.62	−36	1.18	1.07	0.85	0.72	0.68	0.65	0.63	0.61	0.59	(0.59)
10.70	−29	1.33	1.19	0.99	0.88	0.78	0.70	0.61	0.53	0.48	0.45
10.75	−41	1.15	1.05	0.91	0.80	0.74	0.70	0.67	0.66	0.65	(0.64)
10.76	+17	1.07	1.03	0.96	0.87	0.79	0.72	0.67	0.63	0.59	(0.57)
11.00	−65	1.43	1.32	1.11	0.96	0.83	0.76	0.71	(0.68)	(0.67)	(0.67)
11.10	+62	1.40	1.28	1.17	1.03	0.93	0.83	0.75	(0.72)	(0.70)	(0.69)
11.23	+15	1.43	1.31	1.17	1.04	0.97	0.92	0.85	0.79	0.74	(0.72)
11.27	−15	1.37	1.28	1.18	1.06	0.97	0.91	0.88	0.86	(0.85)	(0.84)
11.29	−16	1.33	1.25	1.13	1.03	0.97	0.92	0.89	0.88	0.87	(0.87)
11.33	− 6	1.43	1.32	1.18	1.08	1.01	0.96	0.91	0.86	(0.84)	(0.83)
11.33	−25	1.54	1.45	1.29	1.13	0.99	0.92	0.85	0.80	0.77	0.73
11.33	−38	1.50	1.32	1.15	1.05	1.00	0.95	0.90	0.87	0.85	(0.84)
11.35	−19	1.30	1.24	1.15	1.07	1.01	0.97	0.93	0.90	0.87	(0.86)
11.38	+ 9	1.63	1.47	1.29	1.12	1.01	0.93	0.87	0.82	(0.79)	(0.77)
11.38	−27	1.56	1.46	1.25	1.04	0.97	0.92	0.90	0.87	0.84	0.83
11.38	−34	1.63	1.46	1.27	1.15	1.05	0.96	0.88	0.82	0.76	0.72
11.39	− 1	1.56	1.46	1.29	1.14	1.01	0.92	0.85	(0.81)	(0.79)	(0.79)
11.41	− 5	1.39	1.33	1.23	1.14	1.07	0.99	0.93	0.88	(0.85)	(0.84)
11.43	+14	1.62	1.49	1.32	1.14	1.04	0.97	0.90	0.83	0.77	(0.73)
11.43	− 4	1.42	1.33	1.23	1.15	1.07	0.99	0.94	0.92	(0.91)	(0.90)
11.46	+16	1.51	1.38	1.23	1.12	1.05	1.00	0.95	0.91	0.88	(0.86)
11.48	−18	1.28	1.21	1.13	1.09	1.06	1.03	1.01	0.99	0.97	(0.97)
11.62	− 3	1.47	1.43	1.33	1.20	1.13	1.05	1.01	0.98	(0.97)	(0.96)
11.66	−41	1.65	1.52	1.38	1.23	1.10	1.03	0.99	0.97	0.95	(0.94)
11.71	+22	1.52	1.43	1.33	1.22	1.13	1.07	1.04	1.01	0.99	0.97
11.74	+23	1.47	1.44	1.37	1.29	1.19	1.12	1.04	0.99	0.95	0.93
11.76	−36	1.66	1.48	1.32	1.21	1.14	1.11	1.08	1.05	1.03	(1.02)
11.76	−43	1.66	1.58	1.47	1.34	1.20	1.06	0.99	0.93	0.90	(0.89)
11.80	−42	1.68	1.62	1.50	1.37	1.22	1.10	1.00	0.94	0.88	(0.86)
11.88	−32	1.83	1.67	1.50	1.31	1.17	1.08	1.04	1.03	1.03	(1.03)

TABLE XV—*Continued*

m	Zone	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
12.15	+15°	1.80	1.69	1.54	1.42	1.33	1.24	1.18	1.13	1.08	(1.05)
12.16	−17	1.76	1.63	1.48	1.38	1.31	1.25	1.20	1.17	1.16	(1.15)
12.18	−18	1.69	1.51	1.36	1.33	1.32	1.31	1.28	1.25	1.23	(1.22)
12.19	−19	1.73	1.63	1.47	1.39	1.32	1.26	1.23	1.20	1.17	(1.16)
12.24	+16	1.84	1.72	1.57	1.43	1.33	1.27	1.22	1.18	1.15	(1.13)
12.28	− 6	1.83	1.69	1.53	1.43	1.37	1.32	1.27	1.23	(1.21)	(1.20)
12.32	+62	1.89	1.77	1.67	1.53	1.39	1.27	1.19	(1.14)	(1.11)	(1.10)
12.35	+17	1.80	1.73	1.63	1.48	1.38	1.30	1.25	1.20	1.17	(1.16)
12.43	+28	1.88	1.81	1.63	1.45	1.38	1.35	1.31	1.30	(1.29)	(1.29)
12.43	+14	2.06	1.92	1.69	1.51	1.40	1.33	1.26	1.19	1.13	(1.10)
12.45	−16	1.97	1.83	1.65	1.49	1.37	1.30	1.28	1.27	1.26	(1.26)
12.45	−43	1.97	1.89	1.76	1.60	1.45	1.31	1.21	1.15	1.11	(1.10)
12.47	−15	1.89	1.78	1.64	1.53	1.43	1.36	1.31	1.28	(1.26)	(1.25)
12.48	−29	1.98	1.83	1.68	1.56	1.46	1.37	1.27	1.21	1.18	1.15
12.49	−42	1.95	1.86	1.76	1.63	1.49	1.35	1.25	1.17	1.10	(1.09)
12.51	− 5	1.87	1.76	1.63	1.53	1.44	1.39	1.35	1.32	(1.31)	(1.30)
12.51	−31	2.08	1.89	1.69	1.53	1.39	1.33	1.29	1.27	1.24	1.23
12.51	−34	2.09	1.91	1.69	1.56	1.47	1.38	1.29	1.23	1.17	1.14
12.55	+ 9	2.06	1.94	1.75	1.60	1.47	1.35	1.25	1.20	(1.17)	(1.16)
12.59	− 4	1.96	1.83	1.65	1.53	1.45	1.42	1.38	1.36	(1.35)	(1.34)
12.66	+15	1.99	1.90	1.74	1.63	1.51	1.42	1.34	1.28	1.24	(1.22)
12.74	− 3	1.99	1.90	1.75	1.62	1.53	1.45	1.39	1.36	(1.35)	(1.34)
12.75	+16	2.08	1.95	1.76	1.63	1.52	1.45	1.39	1.34	1.30	(1.28)
12.76	−36	2.00	1.83	1.68	1.58	1.53	1.48	1.44	1.40	1.37	(1.36)
12.79	+22	2.02	1.92	1.77	1.63	1.52	1.45	1.41	1.37	1.33	1.31
12.79	−27	2.13	2.02	1.79	1.63	1.52	1.42	1.38	1.35	1.33	1.32
12.82	+23	1.95	1.90	1.82	1.73	1.59	1.46	1.39	1.34	1.31	1.28
12.84	−18	1.94	1.80	1.64	1.59	1.56	1.53	1.49	1.46	1.43	(1.42)
12.85	−41	2.15	2.03	1.86	1.70	1.55	1.42	1.37	1.36	1.35	(1.35)
12.87	− 6	2.11	1.96	1.78	1.67	1.58	1.50	1.44	1.41	(1.39)	(1.38)
12.88	−25	2.06	1.99	1.83	1.68	1.59	1.51	1.43	1.37	1.33	1.31
12.89	−38	2.08	1.96	1.80	1.65	1.55	1.49	1.45	1.41	1.38	(1.37)
12.92	+14	2.33	2.15	1.91	1.70	1.57	1.48	1.41	1.34	1.29	(1.27)
12.94	− 1	2.24	2.12	1.92	1.72	1.57	1.45	1.38	(1.34)	(1.32)	(1.31)
12.96	−29	2.19	2.06	1.88	1.73	1.62	1.52	1.43	1.38	1.34	1.32
12.97	+62	2.15	2.03	1.90	1.74	1.60	1.51	1.44	(1.39)	(1.36)	(1.35)
13.02	−65	2.26	2.16	1.98	1.76	1.58	1.46	1.40	(1.37)	(1.35)	(1.34)
13.03	−27	2.23	2.13	1.88	1.70	1.58	1.51	1.46	1.43	1.41	1.39
13.04	− 4	2.18	2.03	1.84	1.72	1.59	1.54	1.52	1.49	(1.48)	(1.47)
13.06	− 5	2.18	2.08	1.90	1.76	1.63	1.53	1.47	1.42	(1.39)	(1.38)
13.11	−32	2.32	2.15	1.94	1.76	1.62	1.51	1.46	1.44	1.43	(1.43)
13.18	+17	2.17	2.08	1.93	1.78	1.64	1.58	1.53	1.49	1.47	(1.46)
13.19	−34	2.39	2.18	1.94	1.83	1.71	1.60	1.52	1.45	1.38	1.34
13.21	−31	2.28	2.18	2.02	1.81	1.64	1.54	1.49	1.47	1.45	1.43
13.23	−16	2.43	2.18	1.96	1.78	1.64	1.58	1.53	1.50	1.47	(1.45)
13.25	+23	2.33	2.24	2.08	1.91	1.73	1.56	1.47	1.42	1.39	1.38
13.27	− 3	2.24	2.14	1.98	1.81	1.69	1.62	1.55	1.50	(1.47)	(1.46)
13.27	−15	2.28	2.17	1.99	1.83	1.70	1.59	1.53	1.49	(1.47)	(1.46)
13.30	−19	2.23	2.12	1.93	1.78	1.70	1.63	1.58	1.54	1.52	(1.51)
13.34	+22	2.33	2.22	2.03	1.90	1.75	1.64	1.55	1.48	1.43	1.40
13.36	−25	2.21	2.12	1.97	1.84	1.74	1.65	1.59	1.53	1.50	1.46
13.38	−17	2.29	2.12	1.93	1.82	1.73	1.66	1.60	1.55	1.53	(1.52)
13.38	−18	2.27	2.06	1.87	1.80	1.74	1.70	1.64	1.59	1.56	(1.55)
13.48	−41	2.40	2.26	2.08	1.90	1.75	1.65	1.58	1.55	1.53	(1.53)
13.53	−36	2.38	2.15	1.99	1.85	1.77	1.72	1.68	1.64	1.61	(1.59)
13.54	−43	2.38	2.28	2.15	2.00	1.83	1.67	1.56	1.51	1.46	(1.44)
13.56	−38	2.37	2.20	2.04	1.90	1.80	1.71	1.65	1.60	1.57	(1.57)
13.57	−42	2.36	2.29	2.18	2.02	1.85	1.69	1.59	1.49	1.45	(1.44)
13.68	+28	2.38	2.29	2.10	1.88	1.80	1.75	1.71	1.69	(1.68)	(1.68)

Together, all these data define log N_m as a function of m from 4.0 to 18.5 for each of the latitudes 0°, 10°, ... 90°. The smooth curves obtained by plotting the densities on a large-scale diagram (Fig. 2) eliminate the remaining accidental irregularities from magni-

tude to magnitude for each latitude, and the slight systematic differences between the data drawn from the three sources. The latter are appreciable in latitude 0°, $m = 13.5$ to 15.5, where the densities from the *Mount Wilson Catalogue* are clearly too large (Fig. 2 and Table VII*a*), and in latitudes 80° and 90° in the interval covered by the astrographic zones (Fig. 2). In neither case is the difference surprising. The Selected Areas in and near the Milky Way are few in number, badly distributed in longitude, and show large fluctuations in density. The astrographic counts in this region are far more reliable and really determine, by extrapolation, the 0° curve to

TABLE XVI

MEAN LOG N_m FROM ASTROGRAPHIC ZONES

Mean m	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	No. Zones
8.91.....	0.46	0.36	0.21	0.07	9.97	9.91	9.87	9.84	9.82	9.80	9
9.44.....	0.61	0.52	0.40	0.30	0.22	0.17	0.13	0.09	0.07	0.07	10
10.02.....	0.86	0.78	0.66	0.54	0.46	0.41	0.37	0.34	0.33	0.32	12
10.44.....	1.08	0.98	0.83	0.71	0.63	0.57	0.53	0.50	0.48	0.46	12
11.29.....	1.47	1.36	1.20	1.07	0.98	0.91	0.86	0.82	0.79	0.78	13
11.63.....	1.55	1.45	1.33	1.22	1.12	1.05	0.99	0.96	0.93	0.92	13
12.30.....	1.85	1.74	1.58	1.45	1.36	1.28	1.24	1.20	1.17	1.16	12
12.58.....	2.00	1.86	1.70	1.58	1.47	1.40	1.33	1.28	1.25	1.24	12
12.88.....	2.11	2.00	1.82	1.68	1.57	1.48	1.42	1.38	1.35	1.33	12
13.16.....	2.27	2.14	1.95	1.79	1.65	1.55	1.49	1.46	1.43	1.42	12
13.47.....	2.33	2.19	2.02	1.88	1.77	1.68	1.61	1.56	1.53	1.52	11

$m = 15.5$. In high latitudes, on the other hand, Table XV shows that the densities from the astrographic zones are nearly all extrapolated. The small systematic error, now revealed in the extrapolation, has been corrected in drawing the curves.

To eliminate any irregularity in the spacing of the curves ($\log N_m$ against m for each latitude), ordinates were plotted against galactic latitude for each half-magnitude, 4.0, 4.5, etc. The ordinates of the new series of curves thus formed ($\log N_m$ against latitude for each m) extrapolated to $m = 21.0$, are in Table XVII, which is adopted as representing the mean distribution for the whole sky.

At the end of section 2 it was shown that counts from the *Harvard-Groningen Durchmusterung* for single Selected Areas could be extrapolated from 15.5 or 16.0 to 17.5 without serious error. In

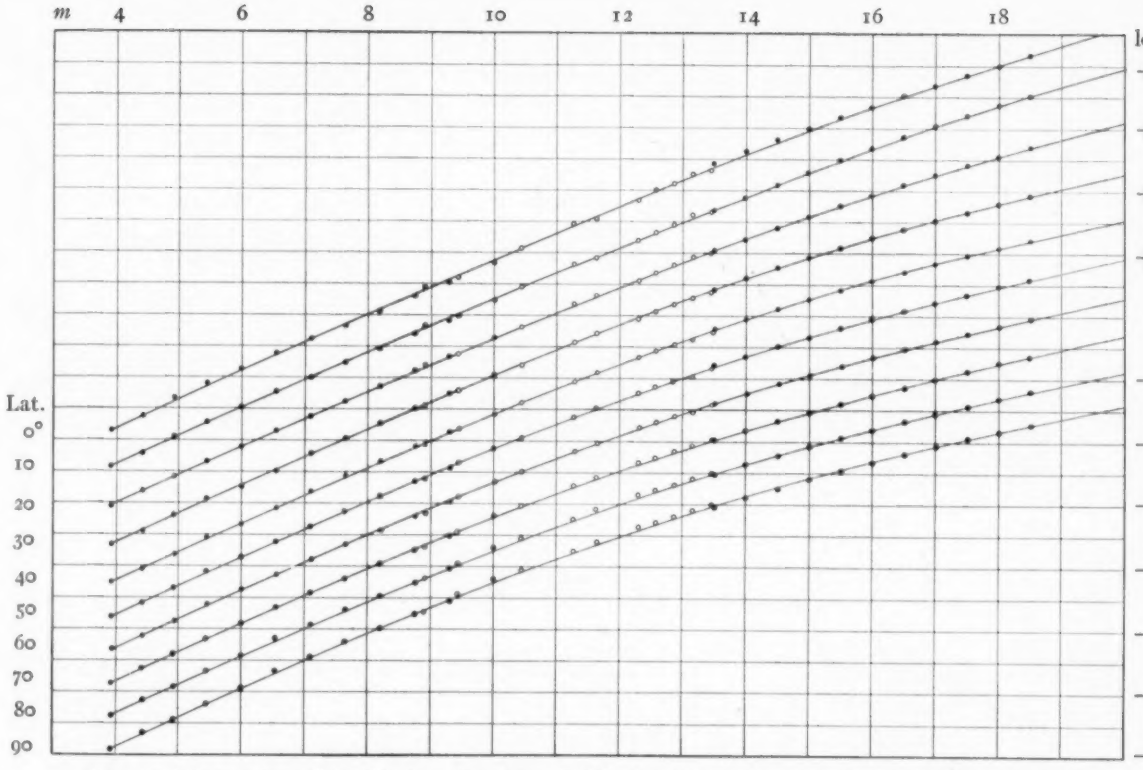


FIG. 2.—Mean values of log N_m from *Groningen Publication*, No. 27 ($m=4$ to 9), from 33 *Astrographic Zones* ($m=9$ to 13.5), and from the *Mount Wilson Catalogue* ($m=13.5$ to 18.5). The curves are the basis of the adopted distribution in Table XVII. The ordinates refer to latitudes 0°. To find log N_m for other latitudes add $0.05\times$Lat. to the printed ordinate.

extrapolating the final results from $m = 18.5$ to 21.0 we are dealing with mean densities based upon a large amount of material. This greatly reduces the danger of systematic error, which is further controlled by the condition that the extrapolated densities must vary smoothly with the galactic latitude. The method used was graphical (see Fig. 2), followed by a numerical smoothing of the differences in $\log N_m$. The greatest danger lies, not in the extrapolation itself, but rather in the magnitude scale in the interval $m = 16.5$ to 18.5. A systematic divergence in scale in this region would of course be carried downward by the extrapolation with steadily increasing effect. Unfortunately, we have no control on the extrapolated result, but it is, at least, an approximation for the distribution of the very faint stars.

6. MEAN VALUES OF LOG N_m

The determination of mean values of $\log N_m$ for wide zones of latitude and for the whole sky requires a summation of the values in Table XVII with respect to galactic latitude. To represent the values of N_m for a given magnitude, Kapteyn has used the formula

$$N_m = A + \frac{B}{1 + h \sin^2 b}, \qquad (1)$$

which is easily integrated with respect to the latitude b; but for the faint magnitudes, at least, the variation in N_m is so large that the interval 0°–90° must be divided into three parts in order to obtain satisfactory results. It is therefore simpler to integrate numerically. This has been done for $m = 4, 5, \ldots 21$ for the intervals 0°–20°, 20°–40°, and 40°–90°, the results being combined to cover the interval 0°–90°. The mean values of $\log N_m$ are in the second to the fifth columns of Table XVIII, data for $m = 4.5, 5.5$, etc., being supplied by interpolation. The last value in the fifth column corresponds to a total, for the whole sky, of 890,000,000 stars to the twenty-first photographic magnitude.

7. MEAN VALUES OF LOG N_m FOR STARS GROUPED ACCORDING TO VISUAL MAGNITUDE

The distribution of stars grouped according to visual magnitude differs appreciably from that given in Table XVII. Detailed results

TABLE XVII

MEAN DISTRIBUTION OF STARS*

m	0°	5°	10°	15°	20°	25°	30°
4.0	8.193	8.168	8.117	8.054	7.989	7.930	7.870
	230	230	230	230	230	230	231
4.5	8.423	8.398	8.347	8.284	8.219	8.160	8.101
	229	229	229	229	229	229	230
5.0	8.652	8.627	8.576	8.513	8.448	8.389	8.331
	228	228	228	228	228	228	229
5.5	8.880	8.855	8.804	8.741	8.676	8.617	8.560
	227	227	226	226	226	227	228
6.0	9.107	9.082	9.030	8.967	8.902	8.844	8.788
	226	226	225	225	225	226	226
6.5	9.333	9.308	9.255	9.192	9.127	9.070	9.014
	225	225	224	224	223	224	225
7.0	9.558	9.533	9.479	9.416	9.350	9.294	9.239
	224	223	222	222	221	222	223
7.5	9.782	9.756	9.701	9.638	9.571	9.516	9.462
	223	222	220	219	219	220	221
8.0	0.005	9.978	9.921	9.857	9.790	9.736	9.683
	222	220	219	218	217	217	218
8.5	0.227	0.198	0.140	0.075	0.007	9.953	9.901
	221	219	217	216	215	215	216
9.0	0.448	0.417	0.357	0.291	0.222	0.168	0.117
	220	218	216	214	213	213	214
9.5	0.668	0.635	0.573	0.505	0.435	0.381	0.331
	219	217	215	212	211	211	212
10.0	0.887	0.852	0.788	0.717	0.646	0.592	0.543
	217	215	213	210	209	209	209
10.5	1.104	1.067	1.001	0.927	0.855	0.801	0.752
	215	213	211	208	208	206	205
11.0	1.319	1.280	1.212	1.135	1.063	1.007	0.957
	214	212	209	206	206	203	201
11.5	1.533	1.492	1.421	1.341	1.269	1.210	1.158
	212	210	207	204	204	200	197
12.0	1.745	1.702	1.628	1.545	1.473	1.410	1.355
	210	208	205	202	200	197	193
12.5	1.955	1.910	1.833	1.747	1.673	1.607	1.548
	208	206	202	199	195	192	188
13.0	2.163	2.116	2.035	1.946	1.868	1.799	1.736
	205	203	200	196	191	187	183
13.5	2.368	2.319	2.235	2.142	2.059	1.986	1.919
	201	200	198	193	186	181	178
14.0	2.569	2.519	2.433	2.335	2.245	2.167	2.097
	197	197	195	190	181	175	172
14.5	2.766	2.716	2.628	2.525	2.426	2.342	2.269
	193	194	191	186	176	170	166
15.0	2.959	2.910	2.819	2.711	2.602	2.512	2.435
	188	190	188	182	172	165	160
15.5	3.147	3.100	3.007	2.893	2.774	2.677	2.595
	183	185	184	177	167	160	154
16.0	3.330	3.285	3.191	3.070	2.941	2.837	2.749
	177	180	179	172	162	154	147
16.5	3.507	3.465	3.370	3.242	3.103	2.991	2.896
	172	174	174	167	157	148	140
17.0	3.679	3.639	3.544	3.409	3.260	3.139	3.036
	167	168	168	162	152	142	134
17.5	3.846	3.807	3.712	3.571	3.412	3.281	3.170
	162	162	162	156	147	136	128
18.0	4.008	3.969	3.874	3.727	3.559	3.417	3.298
	157	156	156	150	142	130	122
18.5	4.165	4.125	4.030	3.877	3.701	3.547	3.420
	152	150	150	144	137	124	116
19.0	4.317	4.275	4.180	4.021	3.838	3.671	3.536
	146	144	144	138	131	118	110
19.5	4.463	4.419	4.324	4.159	3.969	3.789	3.646
	140	138	138	132	125	112	104
20.0	4.603	4.557	4.462	4.291	4.094	3.901	3.750
	135	132	131	126	119	106	98
20.5	4.738	4.689	4.593	4.417	4.213	4.007	3.848
	129	126	124	120	113	100	91
21.0	4.867	4.815	4.717	4.537	4.326	4.107	3.939

* The table gives log N_m: N_m=number of stars per square degree brighter than m, international photographic scale. Pole of Milky Way: α 12^{h}41^{m}20^s, δ+27°21′ (1875).

TABLE XVII—*Continued*

m	30°	40°	50°	60°	70°	80°	90°
4.0	7.870	7.784	7.744	7.711	7.689	7.668	7.655
	231	230	229	229	231	230	230
4.5	8.101	8.014	7.973	7.940	7.920	7.898	7.885
	230	229	228	229	230	230	230
5.0	8.331	8.243	8.201	8.169	8.150	8.128	8.115
	229	228	227	228	228	228	229
5.5	8.560	8.471	8.428	8.397	8.378	8.356	8.344
	228	227	226	227	226	225	226
6.0	8.788	8.698	8.654	8.624	8.604	8.581	8.570
	226	226	225	225	224	223	223
6.5	9.014	8.924	8.879	8.849	8.828	8.804	8.793
	225	225	224	223	221	220	220
7.0	9.239	9.149	9.103	9.072	9.049	9.024	9.013
	223	223	222	221	218	218	217
7.5	9.462	9.372	9.325	9.293	9.267	9.242	9.230
	221	221	220	219	215	215	214
8.0	9.683	9.593	9.545	9.512	9.482	9.457	9.444
	218	219	218	216	212	212	210
8.5	9.901	9.812	9.763	9.728	9.694	9.669	9.654
	216	217	215	212	208	207	205
9.0	0.117	0.029	9.978	9.940	9.902	9.876	9.859
	214	214	211	207	204	202	201
9.5	0.331	0.243	0.189	0.147	0.106	0.078	0.060
	212	210	206	202	199	197	197
10.0	0.543	0.453	0.395	0.349	0.305	0.275	0.257
	209	206	202	197	195	192	192
10.5	0.752	0.659	0.597	0.546	0.500	0.467	0.449
	205	202	198	192	190	187	187
11.0	0.957	0.861	0.795	0.738	0.690	0.654	0.636
	201	198	194	187	184	182	182
11.5	1.158	1.059	0.989	0.925	0.874	0.836	0.818
	197	193	189	182	179	177	177
12.0	1.355	1.252	1.178	1.107	1.053	1.013	0.995
	193	188	183	177	173	172	171
12.5	1.548	1.440	1.361	1.284	1.226	1.185	1.166
	188	183	177	172	168	167	165
13.0	1.736	1.623	1.538	1.456	1.394	1.352	1.331
	183	178	172	167	163	161	160
13.5	1.919	1.801	1.710	1.623	1.557	1.513	1.491
	178	172	166	162	158	156	155
14.0	2.097	1.973	1.876	1.785	1.715	1.669	1.646
	172	166	160	156	152	150	150
14.5	2.269	2.139	2.036	1.941	1.867	1.819	1.796
	166	160	153	150	146	144	144
15.0	2.435	2.299	2.189	2.091	2.013	1.963	1.940
	160	153	146	144	140	138	138
15.5	2.595	2.452	2.335	2.235	2.153	2.101	2.078
	154	146	140	138	134	132	133
16.0	2.749	2.598	2.475	2.373	2.287	2.233	2.211
	147	140	134	132	128	127	127
16.5	2.896	2.738	2.609	2.505	2.415	2.360	2.338
	140	133	128	126	123	122	121
17.0	3.036	2.871	2.737	2.631	2.538	2.482	2.459
	134	127	122	120	118	117	115
17.5	3.170	2.998	2.859	2.751	2.656	2.599	2.574
	128	120	116	114	113	111	109
18.0	3.298	3.118	2.975	2.865	2.769	2.710	2.683
	122	113	110	108	107	105	104
18.5	3.420	3.231	3.085	2.973	2.876	2.815	2.787
	116	107	104	102	102	100	99
19.0	3.536	3.338	3.189	3.075	2.978	2.915	2.886
	110	100	98	96	96	94	93
19.5	3.646	3.438	3.287	3.171	3.074	3.009	2.979
	104	94	92	90	90	88	87
20.0	3.750	3.532	3.379	3.261	3.164	3.097	3.066
	98	87	86	84	84	82	81
20.5	3.848	3.619	3.465	3.345	3.248	3.179	3.147
	91	81	80	78	78	76	75
21.0	3.939	3.700	3.545	3.423	3.326	3.255	3.222

for such a grouping cannot be given at present because of lack of knowledge of the variation of average color-index with galactic lati-

TABLE XVIII

MEAN VALUES OF LOG N_m

m	PHOTOGRAPHIC GROUPING				VISUAL
	0°–20°	20°–40°	40°–90°	0°–90°	0°–90°
4.0	8.116	7.884	7.726	7.938	8.112
4.5	8.347	8.114	7.956	8.168	8.354
5.0	8.576	8.344	8.186	8.398	8.595
5.5	8.804	8.573	8.414	8.626	8.834
6.0	9.030	8.800	8.640	8.853	9.071
6.5	9.255	9.026	8.865	9.078	9.306
7.0	9.478	9.250	9.088	9.301	9.539
7.5	9.700	9.473	9.309	9.523	9.771
8.0	9.920	9.694	9.528	9.743	0.001
8.5	0.140	9.912	9.743	9.961	0.228
9.0	0.358	0.128	9.955	0.178	0.453
9.5	0.575	0.341	0.163	0.392	0.675
10.0	0.791	0.551	0.367	0.604	0.895
10.5	1.005	0.759	0.567	0.814	1.111
11.0	1.217	0.964	0.763	1.020	1.324
11.5	1.427	1.166	0.954	1.224	1.534
12.0	1.634	1.364	1.141	1.425	1.740
12.5	1.839	1.558	1.322	1.623	1.943
13.0	2.042	1.748	1.498	1.818	2.141
13.5	2.242	1.933	1.667	2.009	2.335
14.0	2.439	2.113	1.831	2.196	2.525
14.5	2.633	2.287	1.989	2.379	2.710
15.0	2.823	2.454	2.140	2.559	2.891
15.5	3.009	2.616	2.286	2.734	3.066
16.0	3.190	2.772	2.427	2.905	3.235
16.5	3.366	2.922	2.561	3.071	3.399
17.0	3.537	3.067	2.689	3.232	3.557
17.5	3.703	3.205	2.811	3.388	3.709
18.0	3.864	3.338	2.927	3.539	3.856
18.5	4.019	3.465	3.037	3.684	3.997
19.0	4.169	3.586	3.141	3.824	4.132
19.5	4.313	3.701	3.239	3.959	4.261
20.0	4.450	3.810	3.330	4.089	4.385
20.5	4.581	3.913	3.415	4.214	.....
21.0	4.707	4.011	3.495	4.334	.....

tude and longitude. The relation of mean color to magnitude for all latitudes together, however, was determined in *Contribution* No.

287,[1] where it was found that the correction $-0.16-0.050m$, applied to the argument, transforms a grouping according to photographic magnitude into one according to visual magnitude.

The use of this correction in connection with log N_m for 0°–90° in the fifth column of Table XVIII gives the corresponding mean densities on the basis of a grouping according to visual magnitude. Interpolation of log N_m for corrected magnitudes 4.0, 4.5 . . . . gives at once the results in the sixth column of Table XVIII. The value for the twentieth visual magnitude corresponds to a total of 1,000,000,000 stars for the whole sky.

TABLE XIX

COMPARISON WITH *Draper Catalogue*

Vis. Mag.	*D.C.* Log N_m	Scale Corr.	Color Equation	Corr. Mag.	Table XVIII Log N_m	*D.C. minus* Table XVIII
6.25.....	9.204	0.00	−0.07	6.18	9.156	+0.048
6.75.....	9.434	+ .04	.09	6.70	9.399	+ .035
7.25.....	9.672	.08	.09	7.24	9.650	+ .022
7.75.....	9.891	.12	.10	7.77	9.895	− .004
8.25.....	0.152	.16	.10	8.31	0.142	+ .010
8.75.....	0.379	.21	.11	8.85	0.386	− .007
9.25.....	0.550	+0.26	−0.12	9.39	0.626	−0.076

Comparison with the "Draper Catalogue."—These results can be partially checked by comparison with Shapley's counts of stars in the *Draper Catalogue* given in *Harvard Circular* No. 226, Table II. The totals in the last line of this table, reduced to an area of 1 square degree, give the values of log N_m in the second column of Table XIX. These correspond to magnitudes on the Harvard scale shown in the first column. The scale correction and color equation necessary for reduction to the international system given in the third and fourth columns are taken from Table II and from data under (2) of *Contribution* No. 288. The corrected magnitudes and the corresponding densities interpolated from Table XVIII are in the fifth and sixth columns of Table XIX.

In view of the indirect method of comparison, the agreement shown by the differences in the last column is all that can be expected. The first two or three values are large because of the partial

[1] *Astrophysical Journal*, **61**, 114, 1925.

neglect of the color equation in reducing van Rhijn's table to the international system, which affects the magnitudes near $m=6$ by 0.03 or 0.04 mag. The comparison in Table XIX is not altogether consistent in that the *Draper Catalogue* has been corrected for color. Here, however, the grouping is according to visual magnitude, and

TABLE XX

SCALE CORRECTION FOR KAPTEYN'S DISTRIBUTION TABLE, *G.P.* No. 18

Mag.	Corr.	Mag.	Corr.	Mag.	Corr.
4	−0.17	9	+0.17	14	+0.64
5	− .09	10	.24	15	0.82
6	− .03	11	.31	16	1.07
7	+ .04	12	.39	17	1.41
8	+0.11	13	+0.50	18	+1.88

the relatively large mean color-index of stars grouped in this manner leads to a color equation which cannot be neglected. The last difference in Table XIX is merely an indication that the counts from the *Draper Catalogue* are not complete to the limit in question.

TABLE XXI

COMPARISON OF MEAN COLOR-INDICES

Pg. Mag.	Source	Galactic Latitude					
		0°	10°	30°	50°	70°	90°
7.....	*a**	+0.37	+0.42	+0.54	+0.65	+0.72	+0.76
	b†	0.35	0.43	0.55	0.64	0.68	0.69
14.....	*a*	0.89	0.91	0.75	0.81	0.93	1.02
	b	0.56	0.68	0.84	.....	.....	.....
17.....	*a*	+1.09	+1.02	+0.97	+0.87	+0.66	+0.56

* *a* from distribution tables for stars grouped according to photographic and visual magnitudes.
† *b* from *Contribution* No. 287, data groups (1) and (2), and Table X.

Scale correction to "Groningen Publication" No. 18.—The comparison of the values of log N_m in the sixth column of Table XVIII with those in the last column of Kapteyn's distribution table in *Groningen Publication* No. 18 leads to the scale correction for Kapteyn's table given in Table XX. It should be emphasized that this correction is an average for all galactic latitudes.

Were the correction independent of latitude, the comparison

of Kapteyn's table, reduced to the international system by means of this correction, with Table XVII would throw much light on the variation of mean color-index with galactic latitude. Thus in Table XVII, log N_m corresponds to photographic magnitude m_p. With these values of the density as argument, interpolate the corresponding visual magnitude m_v from Kapteyn's corrected table. The color effect would then be $C=m_p-m_v$, which, by *Contribution* No. 287, is sensibly equal to $\frac{1}{2}(C_v+C_p)$, where C_v and C_p are mean color-indices for groupings of stars according to photographic and visual magnitudes, respectively. Table XXI shows, under *a*, values of C found in this manner. For comparison, means of C_v and C_p from *Contribution* No. 287 are given under *b*. The agreement for $m=7$ is excellent, except at 90° latitude; but for $m=14$ the values *a* increase too slowly, while at $m=17$ the effect of color calculated as above diminishes with increasing latitude, instead of increasing, as is known to be the case. The inference is that Kapteyn's distribution table is not homogeneous in the matter of scale, the correction varying with the latitude.

8. THE DERIVATIVES OF N_m

The values of dN_m/dm, which are of much importance in statistical discussions, have been calculated from the data in Table XVII by

$$\frac{dN_m}{dm}=\frac{N_m}{\text{Mod.}}\frac{d(\log N_m)}{dm}, \tag{2}$$

where, with sufficient accuracy,

$$\frac{d(\log N_m)}{dm}=\log N_{m+\frac{1}{2}}-\log N_{m-\frac{1}{2}}. \tag{3}$$

The results for selected latitudes and intervals of latitude are given in logarithmic form in Table XXII. It is customary to attempt to represent the values of dN_m/dm by quadratic exponentials in m, or the logarithms of the derivatives by ordinary quadratics in m having the form

$$\log\left(\frac{dN_m}{dm}\right)=a+bm+cm^2. \tag{4}$$

Table XXII shows that this will not be possible for the entire range in m, because the second differences are not constant for $m < 12$ or 13. Below this limit, however, the quadratic relation is

TABLE XXII

VALUES OF LOG $\left(\frac{dN_m}{dm}\right)$

m	PHOTOGRAPHIC GROUPING								VISUAL
	0°	30°	60°	90°	0°–30°	20°–40°	40°–90°	0°–90°	0°–90°
4.0	8.219	7.898	7.734	7.680	8.143	7.911	7.753	7.965	8.159
4.5	8.447	8.127	7.963	7.910	8.371	8.139	7.981	8.193	8.400
5.0	8.674	8.355	8.191	8.139	8.598	8.367	8.208	8.420	8.638
5.5	8.900	8.582	8.417	8.364	8.823	8.593	8.433	8.646	8.874
6.0	9.125	8.807	8.641	8.584	9.046	8.818	8.656	8.870	9.107
6.5	9.349	9.030	8.863	8.802	9.268	9.041	8.878	9.092	9.339
7.0	9.572	9.252	9.082	9.016	9.489	9.262	9.098	9.312	9.569
7.5	9.794	9.471	9.299	9.227	9.708	9.482	9.315	9.531	9.797
8.0	0.016	9.688	9.512	9.433	9.926	9.699	9.528	9.748	0.023
8.5	0.236	9.901	9.721	9.634	0.143	9.912	9.736	9.963	0.246
9.0	0.455	0.113	9.924	9.830	0.359	0.123	9.940	0.175	0.466
9.5	0.673	0.323	0.121	0.022	0.574	0.330	0.140	0.384	0.683
10.0	0.889	0.529	0.312	0.209	0.787	0.534	0.336	0.591	0.897
10.5	1.102	0.731	0.498	0.390	0.997	0.736	0.527	0.795	1.106
11.0	1.314	0.928	0.679	0.565	1.204	0.935	0.713	0.995	1.312
11.5	1.525	1.120	0.854	0.735	1.409	1.130	0.894	1.193	1.515
12.0	1.733	1.308	1.024	0.898	1.611	1.320	1.069	1.388	1.714
12.5	1.938	1.491	1.189	1.054	1.812	1.505	1.237	1.580	1.908
13.0	2.141	1.668	1.348	1.205	2.010	1.684	1.398	1.767	2.097
13.5	2.339	1.839	1.502	1.351	2.204	1.857	1.552	1.949	2.281
14.0	2.532	2.003	1.649	1.491	2.394	2.023	1.700	2.127	2.461
14.5	2.720	2.160	1.789	1.625	2.579	2.182	1.841	2.301	2.636
15.0	2.902	2.310	1.922	1.752	2.760	2.334	1.976	2.471	2.805
15.5	3.078	2.453	2.048	1.873	2.936	2.480	2.105	2.636	2.966
16.0	3.248	2.589	2.167	1.988	3.105	2.620	2.227	2.795	3.120
16.5	3.412	2.716	2.279	2.095	3.269	2.754	2.342	2.948	3.269
17.0	3.571	2.836	2.384	2.194	3.427	2.881	2.449	3.095	3.411
17.5	3.725	2.950	2.482	2.287	3.580	3.001	2.549	3.237	3.547
18.0	3.874	3.058	2.573	2.374	3.726	3.115	2.643	3.373	3.677
18.5	4.017	3.159	2.657	2.456	3.865	3.222	2.730	3.502	3.800
19.0	4.153	3.253	2.734	2.531	3.998	3.321	2.808	3.626	3.916
19.5	4.282	3.339	2.803	2.597	4.124	3.413	2.878	3.744	4.026
20.0	4.405	3.417	2.864	2.654	4.242	3.498	2.940	3.857	4.131
20.5	4.522	3.487	2.917	2.702	4.353	3.576	2.995	3.965	.....
21.0	4.631	3.547	2.961	2.743	4.458	3.647	3.042	4.067	.....
a	−5.055	−5.244	−5.179	−5.131	−5.520	−5.185	−5.303	−5.134	−5.010
b	+0.703	+0.715	+0.688	+0.669	+0.744	+0.704	+0.706	+0.6796	+0.7130
c	−0.0115	−0.0141	−0.0143	−0.0140	−0.0128	−0.0135	−0.0147	−0.0115	−0.0128
m_0	30.6	25.4	24.0	23.9	29.1	26.1	24.0	29.6	27.8

applicable. The values of the coefficients a, b, and c are to be found at the bottom of Table XXII, and the corresponding representation, in the sense Table XXII *minus* formula (4) in Table XXIII.

The failure of the quadratic formula for the brighter stars is not unexpected, for the following reason. We have the general relation

$$\frac{dN_m}{dm}\,dm = KdM\int_0^\infty \phi(M)\Delta(\rho)\rho^2 d\rho \qquad (5)$$

where K is a constant, $\phi(M)$ the luminosity function, and $\Delta(\rho)$ the density function. Table III of *Contribution* No. 271[1] shows that practically all the stars brighter than $m=12$ or 13 have absolute

TABLE XXIII

REPRESENTATION OF LOG $\left(\frac{dN_m}{dm}\right)$ BY QUADRATIC FORMULAE

(Table XXII *minus* Eq. (4); unit = 0.001)

m	PHOTOGRAPHIC GROUPING								VIS. GR.
	0°	30°	60°	90°	0°–20°	20°–40°	40°–90°	0°–90°	0°–90°
11	+28	+13	+20	+31	+89	+10	+29	+45	+28
12	+ 8	+ 2	+ 6	+17	+46	+ 1	+17	+23	+11
13	+ 1	0	0	+ 5	+21	− 1	+ 7	+10	+ 1
14	− 1	+ 1	− 1	0	+ 7	− 2	0	+ 1	− 2
15	0	+ 1	− 1	− 2	0	− 3	− 3	− 1	0
16	− 1	+ 3	− 1	− 1	− 2	− 3	− 3	− 1	− 1
17	− 1	0	0	− 2	− 2	0	− 2	0	− 1
18	+ 1	0	+ 1	− 1	+ 1	− 2	+ 1	0	0
19	+ 3	+ 2	+ 3	+ 5	+ 3	+ 4	+ 4	0	0
20	0	+ 1	+ 3	+ 5	+ 2	+ 3	+ 3	− 1	+ 1
21	− 5	− 6	− 2	− 1	− 1	+ 2	+ 2	+ 1	.....

magnitudes brighter than $M=8$. Further, in *Contribution* No. 273[2] it is shown that the luminosities of all such stars can still be represented approximately by a quadratic exponential in M. If now it were possible to represent the derivative in the left-hand member of (5) by a quadratic exponential in m, we should have for $\Delta(\rho)$, according to Schwarzschild's theorem, a quadratic exponential in $\log \rho$, which is impossible, since the space density of stars is not zero for $\rho=0$. Hence, for the bright stars, at least, the assumption of a quadratic exponential for dN_m/dm is not justified.

In view of the probable complexity of the analytic form of the luminosity and density functions, it is, in fact, surprising to find

[1] *Astrophysical Journal*, **59**, 11, 1924. [2] *Ibid.*, **59**, 310, 1924.

that a simple expression for dN_m/dm or its logarithm is applicable at all. That such is the case for the faint stars probably means that the variation of $\Delta(\rho)$ is relatively simple for large values of ρ, so that the quadratic exponential in log ρ applies to a great range in distance, and further that the deviations of the luminosity function from a Gaussian curve do not seriously affect the values of N_m calculated by (5) until a very low limit for m is reached.

9. TOTAL NUMBER OF STARS AND GALACTIC CONCENTRATION

The total of 890,000,000 stars to the twenty-first photographic magnitude, or a round billion to the twentieth visual magnitude, has

TABLE XXIV

COMPARISON OF LOG N_m FOR 0°–90° WITH RESULTS FROM EQ. (6)

m	Cal. Log N_m	Table XVII —Cal.	m	Cal. Log N_m	Table XVII —Cal.
12	1.472	−0.045	17	3.232	0.000
13	1.827	− .009	18	3.539	.000
14	2.196	.000	19	3.824	.000
15	2.558	+ .001	20	4.090	+ .001
16	2.905	0.000	21	4.334	0.000

already been given. The run in the differences in Tables XVII and XXII reflects what has long been accepted as a demonstrated fact, namely, that the total in the system is finite. Attempts to evaluate this total presuppose that the relation between N_m and m defined by the stars within observational reach applies also to the invisible luminous stars. The preceding section suggests how precarious such an assumption must be, but in the absence of any other method of procedure, we make the assumption tentatively, and write for the total in the system

$$N = 41{,}253 \int_{-\infty}^{+\infty} e^{\frac{1}{\text{Mod.}}(a+bm+cm^2)} dm \,. \qquad (6)$$

The fact that (4) does not hold for $m < 12$ or 13 can be disregarded, since the number of stars brighter than this limit is an insignificant percentage of the total. The limiting magnitude, m_0, above which, in accordance with the assumed validity of (6), one-half of the

stars are found, is equal to $-b/2c$. Its values for different galactic latitudes are given in the last line of Table XXII.

The formulae for the evaluation of (6) are well known.[1] As a preliminary, they may be used to test the accuracy of the results for log (dN_m/dm), 0°–90°, in the ninth column of Table XXII by integrating (6) to the upper limits $m=12, 13, \ldots . 21$ (without the numerical factor, however) and comparing the resulting values of log N_m with those in Table XVII. Below $m=13$ the differences, which are shown in the last column of Table XXIV, are insignificant.

Integrating, now to $m=+\infty$, we find

$$N=3.66\times 10^{10}. \tag{7}$$

The total can also be found from the data for a grouping according to visual magnitude given in the last column of Table XXII. The result is

$$N=3.53\times 10^{10}. \tag{8}$$

Further, the zones 0°–20°, 20°–40°, and 40°–90° can be treated separately, the numbers for the zone totals being summed to obtain the total for the system. This gives:

$$\left.\begin{array}{ll} 0°–20° & N_1=2.84\times 10^{10} \\ 20–40 & N_2=0.12\times 10^{10} \\ 40–90 & N_3=0.02\times 10^{10} \\ \hline 0–90 & N\ =2.98\times 10^{10} \end{array}\right\} \tag{9}$$

Results (7), (8), and (9) agree within the uncertainty of the calculation. That under (9) is perhaps the most reliable, and we adopt simply

$$N=3\times 10^{10} \tag{10}$$

as the total number of the stars in the system on the basis of the assumed validity of equation (4).

Finally, by omitting the numerical factor in (6) and applying the formula to the data in the second and fifth columns of Table

[1] See, for example, van Rhijn, *Groningen Publication* No. 27, p. 23.

XXII, we find the numbers of stars per square degree to the limit of the system in latitudes 0° and 90°. The results are

$$N_0 = 5{,}320{,}000; \quad N_{90} = 7160. \tag{11}$$

The corresponding totals to $m = 21$ from Table XVII are

$$N_0 = 73{,}600; \quad N_{90} = 1670. \tag{12}$$

The totals for the three zones under (9) are remarkable in that they assign 95 per cent of all the stars to a region within 20° of the galactic plane, and less than 1 per cent to that third of the sky

TABLE XXV

GALACTIC CONCENTRATION; RATIO OF N_m FOR 0° AND 90°

m	Ratio	m	Ratio	m	Ratio
4	3.4	10	4.3	16	13.2
5	3.4	11	4.8	17	16.6
6	3.4	12	5.6	18	21.1
7	3.5	13	6.8	19	27.0
8	3.6	14	8.4	20	34.4
9	3.9	15	10.4	21	44.2

between 40° and 90° of galactic latitude. The numbers (11) and (12), however, well illustrate the hazardous character of the extrapolation involved in the unrestricted summation of equation (6), for even in the direction of the poles of the Milky Way, less than a fourth of the estimated total number of stars is within reach of observation, while in the Milky Way the fraction is only one-seventieth. Future revisions and extensions of the observational data may therefore produce large changes in these rather speculative results.

The ratio of the numbers (11), which is over 700, emphasizes the enormous galactic concentration for the stars as a whole already suggested in connection with (9). For the interval of brightness covered by the final distribution table, the values of the concentration (ratios of N_m for 0° and 90°) are as given in Table XXV.

10. TOTAL AMOUNT OF STARLIGHT

The total amount of starlight in terms of the light of a star of magnitude 1.0 is easily obtained by a numerical integration based

on the data in the last two columns of Table XXII. Any value of dN_m/dm is sensibly equal to the number of stars in the interval $m-\frac{1}{2}$ to $m+\frac{1}{2}$; and the addition of 0.4 $(1-m)$ to log (dN_m/dm) gives the logarithm of the amount of light contributed by the stars in that interval. For $m>13$, equation (6), with the factor $e^{\frac{0.4}{\text{Mod.}}(1-m)}$ inserted under the integral, can also be used for the calculation. The results per square degree for the successive intervals $m-\frac{1}{2}$ to $m+\frac{1}{2}$ are given in Table XXVI. The amount contributed by the brightest stars is

TABLE XXVI

AVERAGE LIGHT OF STARS PER SQUARE DEGREE

(Interval $=m-\frac{1}{2}$ to $m+\frac{1}{2}$; unit = a star of mag. 1.0, int. scale)

m	Photo.	Visual	m	Photo.	Visual
0	0.00033	0.00038	12	0.00098	0.00204
1	38	050	13	93	200
2	44	062	14	85	182
3	51	075	15	74	160
4	58	091	16	62	132
5	66	110	17	50	102
6	74	129	18	37	076
7	81	148	19	27	052
8	89	166	20	18	034
9	94	186	21	12	021
10	98	200	21.5–∞	0.00016	0.00025
11	0.00099	0.00204	Total	0.01397	0.02647

so small that the extrapolation of log (dN_m/dm) upward to $m=0$ introduces only a small uncertainty.

The totals for the whole sky are:

$$\left.\begin{array}{l}\text{Vis. light} = 1092 \text{ stars of vis. mag. 1.0 int. scale}\\ \text{Pg. light} = 577 \text{ stars of pg. mag. 1.0 int. scale}\end{array}\right\} \quad (13)$$

The first of these values may be compared with results of direct measures of the visual brightness of the sky:

$$\left.\begin{array}{l}\text{Van Rhijn}^1\text{—1440 stars of vis. mag. 1.0 Harv. scale}\\ \text{Yntema}^2\text{—1350 stars of vis. mag. 1.0 Harv. scale}\end{array}\right\} \quad (14)$$

[1] *Groningen Publication* No. 31, 1921. [2] *Ibid.*, No. 22, 1909.

The best indication as to the relation between the units in which the results are expressed is that[1]

$$\text{Harv. mag. } 1.0 = 0.92 \text{ int. scale.} \qquad (15)$$

Allowance for color equation would reduce the value on the right probably by 0.03 or 0.04 mag., so that altogether the discrepancy between (13) and (14) is even larger than appears on the surface.

It is difficult, at present, to say anything conclusive as to the origin of this difference. Table XXVI shows that the stars fainter than the twentieth visual magnitude, which according to the estimate in the preceding section comprise 97 per cent of all the luminous stars, contribute only about 3 per cent of the total amount of starlight. Whatever the uncertainty in number of stars in the system as calculated by (6), it is not likely, therefore, to affect seriously the calculated total of starlight. Since, further, as shown by Table XXVI, the maximum amount of light is contributed by the stars of the eleventh and twelfth magnitudes, it is improbable that the discordance lies wholly in the residual errors in the magnitude scale. The assumption of a constant scale error, which is equivalent to throwing the whole difference into the units, would require the substitution of

$$\text{Harv. mag. } 1.0 = 1.25 \text{ int. scale} \qquad (16)$$

for the probable relation (15), in order to reconcile the results of van Rhijn's measures with those found here.

MOUNT WILSON OBSERVATORY
August 1925

[1] *Mt. Wilson Contr.*, No. 288; *Astrophysical Journal*, **61**, 284, 1925.

SECONDARY STANDARDS OF WAVE-LENGTH IN THE SPECTRA OF NEON AND IRON

By GEORGE S. MONK

ABSTRACT

The primary standard.—The wave-lengths of the primary standard from a Michelson *H*-tube and from two other sources have been compared by means of the interference spectrograph, and the wave-length from one of these, a modified and brilliant tube, has been compared with the wave-lengths of several neon lines recommended by the Committee on Wave-Lengths of the International Astronomical Union.

Neon.—The values obtained for 20 neon lines by compiling the results of a large number of observations on neon indicate that the recommended values for 2 lines, 5852 A and 6402 A, are questionable.

Wave-lengths from the standard iron arc.—In the spectrum of the recommended standard iron arc, 110 lines, including secondary standards between λ 3445 A and 6678 A and certain suggested substitutes for unstable secondaries, have been compared with the spectrum of neon by the interference method. The observed values of the iron lines to the violet of 6000 A agree well with the last observations of Meggers, Kiess, and Burns, while to the red of 6000 A they are systematically about 0.002 A lower than those of the latter observers. Also, there are indications that the values of all the *b* lines are systematically lower in the Pfund arc.

The value of the probable error.—It is believed that the present ratings A, B, and C for the probable errors, as recommended by the International Astronomical Union, should be revised to a more rigorous scale. Further, since with a large number of observations it is possible to obtain a rating of A from values which are not sufficiently consistent to insure accurate reproduction with one or two exposures, it is suggested that some indication should be made in tables of secondary standards of the consistency of the measures for different exposures for a given standard.

The iron arc "in vacuo."—Observational work done thus far on the vacuum iron arc leads to the belief that the adopted system of standards in the spectrum of iron should be based on the arc at atmospheric pressure.

INTRODUCTION

The gradual adoption of the iron arc as a source of secondary standards of wave-length culminated in 1913 in the specification by the International Union for Solar Research[1] of an iron arc of standard dimensions and method of operation. The wave-lengths of the most suitable lines according to the Gale and Adams classification,[2] were measured by several observers with a view to the establishment of tables of secondary and tertiary standards, based on the adopted primary standard, the red cadmium line 6438.4696 A. A comprehensive summary of the work of these investigators is the basis of

[1] *Transactions, International Union for Solar Research*, **4**, 58, 1914.

[2] *Astrophysical Journal*, **35**, 10, 1912.

the report of the Committee on Wave-Lengths of the 1922 meeting of the International Astronomical Union.[1]

One of the recommendations of that Committee was that the light from the iron arc should be taken from the middle 2 mm of a 6-mm vertical arc. Later it was shown by researches at the Mount Wilson Observatory that grave errors could be introduced by the use of so large a portion of the arc,[2] especially in the case of the unstable *c* and *d* lines of the Gale and Adams classification. The measures by St. John and Babcock[3] were consequently made with the light from the middle $1\frac{1}{4}$ mm of a 12-mm Pfund iron arc, except to the red of 6000 A, where a 6-mm arc was used. A comparison of the wave-lengths obtained by them and by other observers showed the necessity of a redetermination of the recommended standards.

At the 1922 meeting of the Union, it was recommended that the following source be adopted:

> . . . . The Pfund arc operated between 110 and 250 volts, with 5 amperes or less, at a length of 12–15 mm used over a central zone at right angles to the axis of the arc, not to exceed 1–$1\frac{1}{2}$ mm in width, and with an iron rod 6–7 mm diameter as the upper pole and a bead of oxide of iron as the lower pole.

The specifications quoted have been followed in the present work.

In order to make this investigation as comprehensive as possible, the writer first made a series of comparisons of the wave-lengths of the primary standard from the Michelson *H*-tube, a cadmium arc *in vacuo* between cadmium-filled carbon poles, and a discharge tube of another form. Then the wave-lengths of a number of neon lines were compared with the primary standard from the last of these sources. Next, the wave-lengths of a selected group of iron lines in the visible region were compared with those of the neon lines, the latter being chosen rather than the primary standard because of ease of operation and greater number of lines.

APPARATUS AND METHODS

The dispersive apparatus, fully described elsewhere,[4] is a Fabry-Perot interferometer with auxiliary dispersion by a concave grating,

[1] *Transactions of the International Astronomical Union*, **1**, 36, 1922.

[2] *Astrophysical Journal*, **42**, 231, 1915; *ibid.*, **46**, 138, 1917.

[3] *Ibid.*, **53**, 260, 1921.

[4] *Ibid.*, **57**, 222, 1923.

and with ordinary precautions is subject to no vibrations or temperature changes which will affect results. The iron arc was mounted so that it could readily be adjusted for focus of the image on the interferometer or for position in vertical and horizontal directions. The neon tube was one chosen from the equipment of Ryerson Laboratory, giving good illumination when operated by a transformer at 20–25 milliamperes. The cadmium tube finally evolved (Fig. 1) was adopted because the ordinary form of tube was found to break down most quickly at the electrodes. In the tube shown, the heating-box is arranged so that the sealed-in electrodes project below, and are not highly heated. While rather more than the usual small amount of cadmium is distilled into the tube, and collected around the electrodes when the tube is exhausted, the pressure remains low because of the lower heat at the electrodes. When operated on a transformed current of a few milliamperes, at a temperature of 315° C., this tube lasts for many exposures, and gives fifty to a hundred times the intensity of the *H*-tube. The lower spaces through which the electrodes pass are exhausted to an X-ray vacuum, and the insealed cup is made thin, and well annealed.

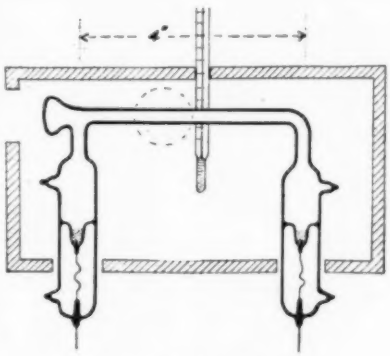


Fig. 1.—Cadmium tube

About one-third of the exposures to the iron arc were made simultaneously to the neon tube, by projecting the light of the latter through the center of the iron arc by means of a concave mirror of appropriate focus. The others were made by alternating the sources every ten to thirty minutes until sufficient exposure had been obtained. The comparisons of the different cadmium sources with one another were made by moving the plateholder sidewise by a push screw between exposures, so that several exposures appear side by side on a single plate. In almost all exposures a Cooper-Hewitt lamp was placed behind the source so that the principal mercury lines, many of which give very sharp fringes for certain

etalons in spite of their complexity, appear as a valuable check on possible changes in the apparatus between exposures. It was found necessary to allow the whole apparatus to rest for several hours after adjustments of the etalon. No spectrograms were used in which these and similar precautions were not taken. The etalon separations were 5, 10, and 15 mm; and for some of the neon plates, 25 mm.

The iron arc, operated at about 5 amp. on a 220-volt circuit, was of the true Pfund type, with the upper pole negative. The diameter of the upper electrode was 7 mm, and a distance of 12–15 mm was maintained between the electrodes. A fivefold image of the source was focused on the aperture of the interferometer, which was made of such a size that it received only the light from the middle $1\frac{1}{4}$ mm of the arc.

The spectrograms were reduced by the formula

$$E+\text{integer}=\frac{D^2}{D_k^2-D_{k-1}^2},$$

where E is the fractional order of interference at the center of a ring system for any line, D is the diameter of any fringe for that line, and $D_k^2-D_{k-1}^2$ is the constant difference of the squares of pairs of contiguous diameters. After the integral parts of the orders of interference were determined by a simple modification of Rayleigh's method, the wave-lengths of the lines were found by comparison with the standard lines by means of the relation

$$2e=p_3\lambda_3=p_1\lambda_1=p_2\lambda_2, \text{ etc.},$$

where the p's and λ's are the orders of interference at the centers of the ring systems and the wave-lengths, respectively, and e is the etalon thickness obtained from the standard lines. Corrections for temperature and atmospheric conditions were made where necessary by the use of the tables published by the Bureau of Standards. A correction-curve for phase dispersion was obtained from the observations, by the use of which corrections to the extent of a few ten-thousandths of an angstrom unit were made to most of the lines.

RESULTS

Cadmium.—The new tube shown in Figure 1 gave the same wave-length for the primary standard as the Michelson *H*-tube. Little can be said in favor of the other lines observed, 5085 A, 4799 A, and 4678 A. The means of about fifteen exposures were in good agreement, but, for instance, the differences between exposures for 5085 amounted to as much as 0.010 A., and a comparison of neon and iron lines, using this line as a standard, gave obviously useless results. This is probably to be attributed to the fact that for strong exposures the satellites of these lines have a marked effect on the measurements. Furthermore, their positions differ with respect to the fringes of the principal component for different etalons. The writer does not believe that these lines are good as standards of wave-length, except perhaps to three decimal places in the mean of a large number of observations.

The arc *in vacuo* gave the same value for the primary standard as the other two sources. But here, also, the probable error was very large, owing to the greater intrinsic width of the line in the arc. In the absence of cadmium or neon tubes or where the observing conditions warrant its use, this source for the primary standard seems to be of considerable usefulness, as is shown by the work of F. L. Brown on titanium.[1]

Since the main object of the present work was to investigate the relation of the neon and iron lines to the primary standard, no additional work was done on the other lines of cadmium.

Cadmium and neon.—A series of nine plates exposed to the new cadmium tube and neon simultaneously gave a value for the primary standard of 6438.4696 ±0.0003 A. The neon lines used as standards in reducing these plates were

5881.895	6266.495	6532.883
5944.834	6334.428	6598.953
6096.163	6506.528	6678.276
6143.062		

Neon.—The spectrum of neon has been photographed with the present apparatus for several purposes during the past four years,

[1] *Astrophysical Journal*, **56**, 53, 1922.

and over fifty plates have been measured. The values for each line, after small corrections have been made for phase dispersion, have been averaged, and appear in Table I. It must be understood that the values in column 3 are not the result of a comparison with the primary standard, but the means of values obtained on the assumption that the Bureau of Standards' table of wave-lengths for neon

TABLE I

WAVE-LENGTHS OF NEON LINES

No. of Obs'ns	P.E.	Monk	B. of S.	Meissner*	Priest†	Wallerath‡	Paschen§	I.A.U. Recommended
46	A	5852.487	.488	.488	.4862	.488	.4875	.488
42	A	5881.895	.895	.896	.8958	.894	.894	.896
47	A	5944.834	.834	.834	.8344	.834	.833	.834
9	A	5975.534	.534	.534	.....	.534	.534	.534
5	B	6029.997	.997	.999	.....	.998	.996	.998
21	A	6074.338	.338	.337	.3383	.337	.335	.338
8	A	6096.163	.163	.163	.1608	.163	.163	.163
35	A	6143.062	.062	.061	.0600	.062	.060	.062
5	B	6163.594	.594	.594	.....	.594	.594	.594
4	B	6217.281	.280	.279	.....	.278	.278	.280
19	A	6266.495	.495	.495	.4948	.495	.497	.495
5	B	6304.788	.789	.788	.7929	.789	.787	.789
37	A	6334.429	.428	.428	.....	.428	.427	.428
29	A	6382.991	.991	.991	.9882	.990	.989	.991
23	A	6402.246	.245	.246	.2392	.246	.246	.....
39	A	6506.528	.528	.527	.....	.528	.528	.528
18	A	6532.883	.883	.881	.....	.882	.879	.882
17	A	6598.952	.953	.953	.....	.953	.954	.953
16	A	6678.276	.276	.275	.....	.276	.275	.276
9	A	6717.043	.043	.042	.....	.042	.044	.042

* *Annalen der Physik*, **51**, 115, 1916.
† *Bulletin, Bureau of Standards*, **6**, 573, 1911.
‡ *Annalen der Physik*, **75**, 34, 1924.
§ *Ibid.*, **60**, 405, 1919.

is, on the whole, correct. This assumption rests in part upon the foregoing comparison of several neon lines with the primary standard. A small number of observations means that the line was not much used as a standard by which to measure other spectra. With the source used for this investigation, none of the neon lines observed has been found to be complex with etalon separations up to 25 mm.

While the results cannot be considered as a redetermination of the wave-lengths in neon, the table is published because it adds something to the existing data. It seems probable that the values

adopted by the International Astronomical Union, 5852.488 and 5881.896, should be modified. The means of all values for the first of these is 5852.4866, and the agreement between individual plates is very good.

Iron.—The results of the measurement of 110 iron lines are shown in Table II. Columns 1, 2, and 3 are self-explanatory. The values in column 5 are taken from the *Bulletin* of the Bureau of Standards (No. 478), and are the results of a redetermination of secondary standards of wave-length in the iron arc. In column 6 are values obtained by Wallerath[1] for certain stable iron lines recommended in Schedule III of the *Report* of the Committee on Wave-Lengths of the International Astronomical Union as substitutes for unsatisfactory secondaries. Columns 7 and 8 are taken from the same report. Column 4 gives the probable error according to the scale recommended by the Committee on Wave-Lengths, where A indicates that the probable error of the result is less than 0.0007, B between 0.0007 and 0.0012, and C that the determination is poor. In the writer's experience, a probable error of as much as 0.0007 for ten to fifteen observations on a line does not indicate a very consistent set of measures, and it is suggested that, if the notation is to be retained, A should represent a probable error of 0.0005. The statement is sometimes made that lines determined with a rating of A are accurately reproducible under the same observing conditions. But since with a sufficient number of observations a rating of A can be obtained for almost any line, however inconsistent the individual measures may be, it is suggested that future tables of recommended standards be accompanied by a statement as to the consistency to be expected for each line. To be sure, this is in a sense implied by the classification of the lines according to their stability and freedom from pole-effect, but additional evidence as to the value of the collected data would be given by publishing the probable error of a single observation, or some other mark of the consistency of the measures. Accordingly, the actual probable errors of the observations are included in Table II, column 9, and from these values the probable errors of single observations can be obtained.

[1] *Annalen der Physik*, **75**, 34, 1924.

TABLE II

Wave-Lengths of Lines from the Standard Iron Arc

Intensity Group and Class	λ Monk	No. of Observations	Prob. Error	*B.S. Bull.* 478	Wallerath	Sec. Stds.	Interpolated	P.E.
4	3445.152	7	B	.151	.....	.154	.153	±.0011
6	3485.342	6	B	.343	.....	.345	.343	.0007
5	3513.822	14	A	.821	.....	.821	.821	.0004
5	3556.882	13	A	.882	.....	.881	.881	.0004
5	3558.517	17	A	.517	.520	.....	.519	.0003
5	3606.683	17	A	.683	.....	.682	.682	.0003
6	3640.393	17	A	.393	.....	.392	.392	.0003
4	3676.314	15	A	.315	.....	.313	.313	.0003
6	3677.631	17	A	.631	.....	.629	.629	.0003
6	3724.381	16	A	.381	.....	.380	.380	.0004
5	3753.615	13	A	.615	.....	.615	.615	.0004
6	3805.346	17	A	.346	.....	.346	.346	.0003
5	3843.261	17	A	.261	.....	.261	.261	.0003
5	3850.821	16	A	.....	.....	.820	.821	.0004
6	3865.528	15	A	.526	.....	.527	.527	.0004
5 *a* 1	3906.483	17	A	.483	.....	.482	.483	.0004
3 *b*	3907.937	15	A	.938	.....	.937	.937	.0004
4 *b*	3935.815	10	A	.816	.....	.818	.817	.0005
4 *a*	3940.882	4	A	.....	.....	.884	.884	.0004
5 *b* 4	3977.744	17	A	.744	.....	.746	.746	.0001
5 *b*	4021.870	17	A	.870	.....	.872	.872	.0002
3 *d*?	4074.788	3	A	.789	.794	.....	.792	.0005
5	4076.637	13	A	.636	.....	.642	.638	.0003
3 *b*	4095.972	13	A	.974	.....	.....	.977	.0003
5 *b*	4107.491	15	A	.492	.494	.....	.495	.0003
6 *b*	4118.549	15	A	.549	.....	.552	.552	.0001
5 *b* 4	4134.680	16	A	.680	.....	.685	.684	.0002
4 *b*	4147.673	14	A	.673	.....	.676	.675	.0002
4 *b*	4156.803	13	A	.802	.805	.....	.805	.0005
4 *b*	4175.638	15	A	.639	.642	.....	.639	.0003
4 *b*	4184.894	14	A	.894	.896	.....	.897	.0003
6 *d*	4191.435	17	A	.436	.....	.443	.436	.0002
3 *b* 3	4203.987	16	A	.986	.989	.....	.986	.0004
5 *b*	4219.364	17	A	.364	.365	.....	.367	.0002
6 *d* 5	4233.609	18	A	.609	.....	.615	.614	.0004
2 *b*	4245.260	15	A	.260	.262	.....	.261	.0003
6 *b* 1	4282.406	20	A	.406	.....	.408	.408	.0003
5 *b* 3	4315.087	17	A	.087	.....	.089	.090	.0003
4 *b* 3	4352.737	9	A	.738	.....	.741	.740	.0002
5 *a* 3	4375.933	10	A	.932	.....	.934	.934	.0002
5 *a* 3	4427.313	11	A	.312	.....	.314	.314	.0003
5 *b* 4	4466.556	11	A	.555	.....	.556	.556	.0003
5 *c* 4	4494.568	11	A	.568	.....	.572	.571	.0002
5 *b* 3	4531.152	15	A	.152	.....	.155	.155	.0002
3 *b*	4547.850	15	A	.851	.....	.853	.854	.0002
4	4592.655	14	A	.655	.....	.658	.657	.0002
4	4602.945	14	A	.945	.....	.947	.946	.0001
4 *b*	4647.437	13	A	.437	.....	.439	.439	.0002
4 *c* 4	4691.414	14	A	.414	.....	.417	.417	.0003

TABLE II—*Continued*

Intensity Group and Class	λ Monk	No. of Observations	Prob. Error	*B.S. Bull.* 478	Wallerath	Sec. Stds.	Interpolated	P.E.
5 *c* 5	4707.280	14	A	.282	.....	.288	.288	± .0003
3 *b*	4710.286	11	A	.287	.288	.....	.....	.0002
3 *a*	4733.596	11	A	.596	.598	.....	.598	.0002
5 *c* 5	4736.780	12	A	.782	.....	.786	.790	.0002
3 *b*	4741.531	10	A	.533	.534	.....	.535	.0004
3 *b*	4772.817	10	A	.818	.818	.....	.818	.0004
3 *b*	4789.655	15	A	.654	.....	.657	.656	.0002
5 *c* 5	4859.748	17	A	.748	.....	.758	.759	.0002
5 *c* 5	4878.219	15	A	.219	.....	.225	.224	.0004
5 *c* 5	4903.317	15	A	.317	.....	.325	.326	.0003
8 *c* 5	4919.000	11	A	.001	.....	.007	.008	.0005
3 *a*	4924.776	10	A	.775	.777	.....	.776	.0004
3 *a*	4939.689	11	A	.692	.692	.6	.691	.0002
5 *c* 5	4966.098	15	A	.097	.....	.104	.106	.0004
3 *a*	4994.132	11	A	.132	.136	.....	.135	.0003
5 *c*	5001.871	14	A	.872	.....	.881	.881	.0004
4 *a*	5012.071	14	A	.072	.....	.073	.073	.0002
4 *a*	5041.759	11	A	.758	.760	.....	.760	.0005
5 *a*	5049.825	12	A	.825	.....	.827	.827	.0003
4 *a*	5083.342	12	A	.343	.....	.344	.343	.0004
4 *a*	5110.414	12	A	.414	.....	.415	.415	.0002
4 *a*	5123.723	8	A	.722	.7.6	.....	.725	.0006
4 *a*	5150.843	9	B	.843	.844	.....	.845	.0010
8 *a*	5167.491	14	A	.490	.....	.492	.493	.0005
8 *d*	5192.351	14	A	.353	.....	.363	.360	.0004
5 *a*	5202.339	9	A	.339	.341	.....	.340	.0005
5 *a*	5216.277	7	B	.277	.280	.....	.280	.0010
8 *d* 5	5232.948	14	A	.948	.....	.957	.956	.0006
3 *a*	5250.650	5	B	.650	.651	.652	.652	.0010
8 *d* 5	5266.563	12	A	.564	.....	.569	.571	.0005
8 *a* 4	5270.360	8	A	.361	.361	.....	.360	.0003
5 *d*	5302.309	7	A	.308	.....	.315	.315	.0005
6 *d* 5	5324.187	12	A	.187	.....	.196	.196	.0004
4 *a* 4	5328.534	12	A	.532	.537	.537	.537	.0004
5 *a* 4	5341.026	12	A	.026	.027	.....	.028	.0004
7 *a* 1	5371.493	16	A	.493	.....	.495	.495	.0003
6 *a* 4	5405.779	12	A	.778	.....	.780	.780	.0003
6 *a* 4	5434.527	17	A	.527	.....	.527	.528	.0002
6 *a* 4	5455.612	17	A	.613	.....	.614	.615	.0002
4 *a* 3	5497.519	17	A	.520	.....	.522	.522	.0003
4 *a* 3	5506.782	17	A	.783	.....	.784	.784	.0004
5 *d* 5	5569.625	15	A	.627	.....	.633	.....	.0003
7 *d* 5	5572.850	16	A	.....	.....	.....	.....	.0003
6 *d* 5	5586.764	11	A	.764	.....	.772	.....	.0003
6 *d* 5	5615.652	12	A	.653	.....	.661	.....	.0006
5 *d* 5	6024.064	3	C	.066	.....	.....	.....	.0014
2 *b* 4	6027.058	6	A	.058	.....	.059	.059	.0006
4 *b* 4	6065.488	14	A	.489	.....	.492	.492	.0003
4 *b* 4	6136.621	16	A	.623	.....	.....	.624	.0003
4 *b* 4	6137.695	16	A	.697	.....	.701	.702	.0004
5 *b* 4	6191.562	22	A	.564	.....	.568	.568	.0004

TABLE II—*Continued*

Intensity Group and Class	λ Monk	No. of Observations	Prob. Error	*B.S. Bull.* 478	Wallerath	Sec. Stds.	Interpolated	P.E.
5 *b* 4	6230.728	21	A	.730	.....	.734	.734	±.0002
3 *b* 4	6265.141	8	B	.140	.....	.145	.145	.0007
4 *b* 4	6318.022	9	A	.024	.....	.028	.028	.0005
4 *b* 4	6335.335	7	B	.338	.....	.341	.342	.0010
5 *b* 4	6393.604	30	A	.607	.....	.612	.612	.0004
5 *b* 4	6430.851	25	A	.853	.....	.859	.859	.0004
5 *b* 4	6494.985	34	A	.987	.....	.993	.993	.0003
5 *b* 4	6546.246	12	B	.247	.....	.252	.252	.0009
5 *b* 4	6592.920	15	A	.920	.....	.928	.927	.0006
5 *b* 4	6677.991	11	B	.994	.....	8.004	.001	±.0009

Five of the exposures of the region to the violet of 5000 A were made alternately with exposures to the neon lines and iron lines to the red of λ 4200. The etalon separations for these were derived from the diameters of fringes for a number of stable lines in the region common to the two settings, and from the mercury lines. All other exposures to the violet region were made in the second order of the auxiliary spectrograph with the neon lines appearing on the same plate in the first order. The magnification of the spectrograph was a constant for this setting, as for all others. Since there was no systematic difference between the values of the iron lines obtained by the two methods, they are combined in the table.

DISCUSSION

It will be seen from Table II that there is no appreciable systematic difference between the wave-lengths published in this paper and those from Bureau of Standards, *Bulletin* 478, except in the red, where the writer's values are systematically about 0.002 A lower. The arc used at the Bureau of Standards, while complying in all other respects with the recommendations of the Committee on Wave-Lengths, was positive at the upper pole and negative at the lower. The recommended Pfund arc has the opposite polarity. The direction of the current certainly makes a radical difference in the appearance and character of the arc, and it is possible that the difference in behavior is the cause of the observed differences in wave-lengths. Table III shows a comparison of lines of the same class for all lines classified.

This comparison may be taken to indicate that for the Pfund arc the wave-lengths of the *b* lines are systematically lower than the wave-lengths obtained with the polarity reversed, while the *a* lines seem to be the same in the two arcs. If this effect on the *b*-lines is of the same nature as pole-effect, the greater difference at the red end is explained.

In a recent contribution, Babcock[1] notes that he has obtained systematically lower values than the Bureau of Standards for the red *b* lines of about 0.003 A. It is perhaps worth mentioning that

TABLE III

BUREAU OF STANDARDS −MONK=+Δ

CLASS	NUMBERS OF LINES			
	Total	+Δ	Δ=0	−Δ
a.....	27	7	12	8
b.....	41	23	15	3
All lines classified..	90	39	35	6

while checking the phase correction for a series of measurements of manganese lines in 1922, the writer obtained the following values for four iron lines:

			Plates
6191.561	±	0.0009.....	6
6230.728	±	0.0011.....	6
6393.606	±	0.0010.....	6
6494.985	±	0.0005.....	7

These agree well with the values in Table II.

Aside from the apparent difference in the wave-length of the *b* lines from the arcs with opposite polarities, it appears to the writer that the principal advantage in having the upper pole negative is the greater stability and convenience in operation. Shielded from draughts, and properly trimmed and adjusted before operation, the Pfund arc as specified will go steadily for an hour or more with no attention.

Of the lines suggested as substitutes for unstable secondaries, 5328.534 is regarded as poor, except perhaps for the highest resolv-

[1] *Physical Review*, **25**, 716, 1915.

ing-power. There is another line of greater intensity at 5328.06 A which undoubtedly affects the measures.

A comparison of columns 2 and 6 shows that the values of Wallerath, while agreeing with the adopted standards in column 8, are about 0.002 A higher than those of the Bureau of Standards and of Monk. There is an apparent tendency on the part of the last-mentioned observers to run about 0.001 A higher than the older system of secondaries for the region 3400–3900 A.

It has been the opinion of many engaged in the measurement of standards of wave-length in the spectrum of iron that the system of secondary standards in iron should be based on the arc in a vacuum. With this belief, and in accordance with the obvious need for such determinations, as indicated by the work of Meggers and others on regularities in spectra, the writer included in his program the determination of wave-lengths from the vacuum arc. Several plates have been taken and some measured. Although this part of the work has not advanced sufficiently for the publication of specific details, enough has been done to warrant the following remarks.

1. A system of secondary standards based on the vacuum iron arc would be of little use to many observers who have not such equipment available, particularly since the present determinations of pole-effect and pressure shift are not sufficiently accurate for use in connection with such tables.

2. The use of the iron arc in vacuum as a standard of comparison is not to be recommended because of its lack of simplicity, and its faintness as compared with the arc in air.

3. Such observations as have been made at this Laboratory indicate that the wave-lengths from the vacuum arc lack consistency to about the same extent as those from the arc in air.

RYERSON PHYSICAL LABORATORY
May 1925

ON THE STRUCTURE AND ORIGIN OF SOLAR MAGNETIC FIELDS[1]

By SVEIN ROSSELAND[2]

ABSTRACT

Discussion of solar magnetism on the basis of ordinary electrodynamics and electron theory.—The *first section* of the investigation *discusses the observations* of the general magnetic field. It is pointed out that the current picture of the general magnetic field as being similar to that of an elementary magnet in each level, while the field-strength varies rapidly from level to level, is irreconcilable with ordinary magnetic theory. An alternative, not suffering from this disadvantage, is discussed. The *second section considers the origin* of the magnetic fields. It is shown that the motion of an ionized gas must, in general, be accompanied by an electric current, *even if, macroscopically, the gas is electrically neutral.* This effect, as well as others, appears to be much too small to account for the large values of the field found by observation. If the solar substance emits long-range β- or α-particles, the situation would be more favorable for an explanation, although we should have to contemplate very extreme conditions in order to explain solar magnetism in this way.

Through the extensive work of Hale and his collaborators at the Mount Wilson Observatory it has become an established fact that sun-spots are the seat of intense magnetic fields the strength of which may amount to several thousand gausses. The magnetic field in a spot is generally centrally symmetrical around an axis perpendicular to the sun's surface, and coinciding with the center of the spot. Judged from an investigation by Hale, Ellerman, Nicholson, and Joy,[3] the detailed structure of the field does not seem to deviate much from that of a single magnetic pole, or from the field corresponding to the superposition of a small number of such poles with parallel axes. This would mean, in general, that the electric current generating the field must circulate in approximately circular paths with the spot as a center. A complete investigation of the spatial distribution of magnetic intensity might give important information about the localization of this current; but, for the moment, observation does not appear to permit more definite conclusions on this point.

[1] *Contributions from the Mount Wilson Observatory*, No. 302.

[2] Fellow of the International Educational Board 1924–26.

[3] *Mt. Wilson Contr.*, No. 165; *Astrophysical Journal*, **49**, 153, 1919.

In addition to these local fields, the sun possesses also a general field, symmetrical around an axis which is but slightly inclined to the main axis of the sun's rotation. The strength of this latter field appears to average about 40 gausses just above the photosphere and to decrease so rapidly with increasing height as to be less than 10 gausses at a distance of 500 km above the photosphere.[1] Although the available observations are scarcely accurate enough to warrant definite conclusions to be drawn about the detailed distribution of the general field over the solar surface, we shall discuss this question a little more closely in the following section.

Constitution of the general magnetic field of the sun.—The observations on the general magnetic field of the sun are based on measurements of the Zeeman effect of solar lines. Because of the feeble intensity of the field and the width of the lines, it has in no case been possible to observe separately the magnetic components into which a single line is split up in the presence of the field. Under the particular observational conditions at hand, however, the superposed intensity of all components will not have its maximum at the same place as the line without the field, but will be displaced by small amounts proportional to the component of the magnetic force in the line of sight.[2] Under strictly stationary conditions it would thus be impossible by such observations to determine all components of the magnetic field. In the case of the sun, however, the determination of all components is possible, in principle, because the earth is moving around the sun, and the sun rotating around an axis which makes a finite angle with the perpendicular to the plane of the ecliptic. It can scarcely be doubted, however, that the observations do not determine all components with the same precision, as the magnetic field is found to be axially symmetrical, with its axis only very little inclined to the axis of rotation of the sun, while this latter is but little inclined relative to the axis of the ecliptic. In the limit, when all three axes are parallel, the component of the magnetic field which is parallel to the axis of symmetry would practically escape observation, and it follows that for small values of the inclinations the determination of this

[1] G. E. Hale, F. H. Seares, A. van Maanen, and F. Ellerman, *Mt. Wilson Contr.*, No. 148; *Astrophysical Journal*, **47**, 206, 1918.

[2] Cf. F. H. Seares, *Mt. Wilson Contr.*, No. 72; *Astrophysical Journal*, **38**, 99, 1913.

component becomes especially difficult. This difficulty can be removed very simply by the application of general magnetic theory.

Denoting the magnetic field-strength by H (considered as a vector) this theory requires that

$$\text{div } H = 0 \tag{1}$$

under all conditions, provided a small correction for the possibly finite value of the magnetic susceptibility of the substance in question may be neglected. Under stationary conditions the equation

$$\text{curl } H = 4\pi I \tag{2}$$

is also valid, I being the electric current-density. Assume the current generating the magnetic field to be situated within a finite portion of space in the vicinity of the origin of co-ordinates, a condition which corresponds to the case of the sun. When the x- and y-components of the field are known, we obtain the z-component from (1) in the form

$$H_z = {}^{\infty}_{z}\!\int (\partial H_x/\partial x + \partial H_y/\partial y)\,dz + \alpha\,, \tag{3}$$

where α is an arbitrary function of x and y. The magnetic force must, however, be zero at an infinite distance from the system, and we are therefore compelled to put $\alpha = 0$ everywhere. The component parallel to the axis of symmetry is therefore given uniquely as soon as the component perpendicular to this axis is known. On the other hand, when the observations are extended to all three components, the foregoing considerations should afford a useful check on the interpretation of the observations. Thus the measurements of the solar magnetic field are reported to be compatible with the view that this field, measured in a particular level, is similar to that of an elementary magnet situated at the sun's center with its axis directed nearly parallel to the sun's axis of rotation. Ascending from lower to higher levels, the field intensity decreases very rapidly.[1] The components of the magnetic force would then be given by the expressions

$$H_x = xzA_r\,; \qquad H_y = yzA_r \tag{4}$$

$$H_z = -(x^2+y^2-2z^2)A_r/3\,; \qquad r^2 = x^2+y^2+z^2\,, \tag{5}$$

[1] Cf. G. E. Hale, F. H. Seares, A. van Maanen, and F. Ellerman, *Mt. Wilson Contr.*, No. 148; *Astrophysical Journal*, **47**, 206, 1918.

where A_r is a function of r which decreases very rapidly with increasing height above the photosphere. By (1), (4), and (5) we obtain

$$\text{div } H = 2\varkappa/3r^4 \frac{\partial}{\partial r}(r^5 A_r) . \tag{6}$$

This expression vanishes only for $r^5 A_r = \text{constant}$, corresponding to the case of an elementary magnet, which in turn would correspond only to an insensible change in the magnetic field from level to level in the solar atmosphere. The requirements of equation (1) thus necessitate a reinterpretation of the observational results; either the conclusion that the magnetic field-strength decreases rapidly with increasing height in the atmosphere must be given up, or the similarity of the field in a particular level to that of an elementary magnet must be doubted.

There seems to be a way to escape from this dilemma without violating the results of observations. On account of the small inclination of the axis of symmetry to the axis of the ecliptic, the observations are essentially concerned with the x- and y-components only, while the z-component enters the problem in terms having coefficients so small that it probably does not influence the result materially. The component H_z is thus practically outside of direct observational control, and we are at liberty to adjust its value in such a manner that equation (1) is satisfied. In this way we obtain for H_z, on using (3) and (4), the expression

$$H_z = -(x^2+y^2)A_r + 2\int_z^\infty A_r z dz . \tag{7}$$

If h denotes a certain length which is a function of r and z and of the order of magnitude of the height of the solar atmosphere, equation (7) may be written in the form

$$H_z = -(x^2+y^2-h^2)A_r . \tag{8}$$

Except for a region close to the poles, h^2 must be negligibly small compared to x^2+y^2. A comparison between (4) and (7) shows, therefore, that at the magnetic equator the corrected field-strength is about three times as large as that given by (5), and that the lines

of force run nearly parallel to the solar surface within the whole region covered by observation.

If the measurements of the magnetic field of the sun in different levels could be made with great precision, it would be possible, by application of equation (2), to decide whether the solar atmosphere is the seat of electric currents or not. This analysis, applied to the magnetic field of the earth, leads, as is well known, to the result that the currents generating this field are localized, primarily, in the earth's interior. In the case of the sun conditions seem to differ essentially from those on the earth; and if the strong decrease of magnetic field-strength with increasing height in the solar atmosphere be accepted as an observational fact, strong electric currents must necessarily exist in the atmosphere. Assume, as an illustration, the general magnetic field to have the constitution determined by (4) and (8). By (2), we then obtain the following expressions for the rectangular components of the current density

$$I_x=-I_0 y/r\,; \quad I_y=I_0 x/r\,; \quad I_z=0\,; \quad 4\pi I_0=r^2\partial A_r/\partial r+rA_r\,. \tag{9}$$

The current thus flows along the magnetic parallels, the current density being given by

$$I=I_0\cos\theta\,, \tag{10}$$

where θ is the magnetic latitude of the point in question. The conditions are thus as though the current were produced by uniform rotation of a spherically symmetrical distribution of electricity in the atmosphere. The dominating term in the expression of I_0 is negative, and it follows that the electron current flows in the direction of rotation of the sun. The magnetic field due to a uniformly charged rotating sphere corresponds to that of an elementary magnet outside the charged surface, and is constant in strength and direction (parallel to the axis of rotation) inside the surface. The field given by (4) and (6) is, however, essentially different from the field in the electric layer of a uniformly charged, rotating sphere; and the foregoing results would mean, if taken literally, that the solar field is largely generated by electric currents which are localized below the reversing layer. The precision of the observational results is, how-

ever, scarcely great enough for conclusions of this kind to be warranted for the moment.

Origin of electric currents producing solar magnetism.—Apart from such minor uncertainties as are discussed in the preceding section, the existence and general character of the solar magnetic fields are established beyond doubt. The origin of the fields, on the other hand, appears to remain essentially in the dark, and in this section we shall take up a general discussion of this question, on the basis of ordinary electromagnetic theory as applied to free electric charges. The result of the investigation turns out to be essentially negative, but it is hoped that a detailed discussion of the difficulties may serve a useful purpose.

A stationary magnetic field is, according to ordinary electromagnetic theory, always connected with a system of steady electric currents, and when the current-density is known throughout all space it is a simple process of integration to obtain the corresponding magnetic field. The possible kinds of steady electric currents are conveniently subdivided into three different classes: intermolecular, convectional, and galvanic. Solar magnetism might be assumed to be produced by currents belonging to any of these classes. It appears, however, that the first two hypotheses may be eliminated. In fact, from known values of the atomic magnetic moments of different substances, combined with a liberal estimate of the gas density in the solar photosphere, it can be concluded that it is virtually impossible to account for magnetism in sun-spots by assuming the atoms of the solar gases to be paramagnetic and to be all arranged parallel to a given direction. Moreover, this hypothesis ignores the fact that the systematic arrangement of the atoms would, in absence of other causes, have to be brought about by rotation as in the well-known experiments of S. J. and L. J. Barnett; but the rotational velocity of the sun as a whole, as well as that presumable for the gases in the spots, is so small that any perceptible effect of the kind anticipated must be regarded as definitely excluded. The same argument may be raised against the hypothesis that solar magnetism is generated by convection currents. Assume, for instance, the sun's general field to be generated by rotation of a uniform volume

charge. This charge would correspond to an electric field intensity at the sun's surface of the order 10^6 volts per centimeter, and if this field were for a moment conceived as existing, the solar atmosphere would emit surplus ions with a rapidity which can be characterized best as an explosion. We are therefore forced to admit that solar magnetism must somehow owe its origin to galvanic currents, i.e., to a real interdiffusion of free electrons through the mass of positive ions.

For a closer discussion of this point it is convenient to consider the question from the point of view of diffusion theory, using for this purpose not the complicated, exact methods of Chapman and Enskog, but a short-cut method which more directly reveals the underlying physical phenomena. Consider a gaseous mixture of free electrons and positive ions moving in any manner. We shall write the hydrodynamical equation for the motion of the electrons parallel to the x-axis in the form

$$n_e m_e du_e/dt = -\partial P_e/\partial x - n_e \partial \phi_e/\partial x - \Gamma_x^e , \qquad (11)$$

the equations for the motion along the other axes being of exactly the same form. In the foregoing equation m_e denotes the mass of an electron, n_e the number of electrons per unit volume, u_e the component of the arithmetical mean velocity of the individual electrons at the point x, y, z, at the time t, while the y- and z-components will be denoted by v_e and w_e; p_e is the hydrostatic electron pressure which is given by $n_e kT$ where k is Boltzmann's constant, and T the temperature of the gas; ϕ_e is the potential of the external force acting on a single electron, while Γ_x^e is the x-component of the aggregate frictional resistance experienced by the electrons in unit volume. The equations of motion for the atoms will be of exactly similar form, and will be distinguished by an index a. Each frictional term is conveniently subdivided into two different parts, Γ_{x1}^e and Γ_{x2}^e, each of which proceeds from a specific source. Thus Γ_{x1}^e will be assumed to correspond to the viscosity which according to ordinary gas theory corresponds to a homogeneous gas, the expression of which is

$$\Gamma_{xx}^e = \partial \Gamma_{xx}^e/\partial x + \partial \Gamma_{xy}^e/\partial y + \partial \Gamma_{xz}^e/\partial z ,$$

where

$$\Gamma^e_{xx}=-2\kappa_e\frac{\partial u_e}{\partial x}+\tfrac{2}{3}\kappa_e\left(\frac{\partial u_e}{\partial x}+\frac{\partial v_e}{\partial y}+\frac{\partial w_e}{\partial z}\right);\quad \Gamma^e_{xy}=-\kappa_e\left(\frac{\partial v_e}{\partial x}+\frac{\partial u_e}{\partial y}\right);$$
$$\Gamma^e_{xz}=-\kappa_e\left(\frac{\partial w_e}{\partial x}+\frac{\partial u_e}{\partial z}\right), \tag{12}$$

and where κ_e is a special coefficient of viscosity which will be considered more closely later on. The second term Γ^e_{x2} refers to the additional interaction between electrons and atoms, which is due to the fact that the mean motions of the two different kinds of particles are in general not identical. It is easy to see that Γ^e_{x2} must depend essentially on the difference in mean velocity of electrons and atoms, as it must disappear when these velocities are equal. It can, moreover, be shown that to a high degree of approximation Γ^e_{x2} will be proportional to a certain linear average value of the x-component of the relative velocity only.[1] The average in question will only in special cases be identical with an ordinary arithmetical average, but for the present purpose this distinction will be neglected by writing

$$\Gamma^e_{x2}=\beta_e(u_e-u_a), \tag{13}$$

where β_e is an additional coefficient of viscosity.

In an ordinary gas consisting of atoms which are electrically neutral, it offers no difficulty to derive approximate expressions for the coefficients κ and β. In fact, by ordinary gas theory, κ is given by

$$\kappa=\tfrac{1}{3}\,nm\lambda c, \tag{14}$$

where c is the mean thermal velocity of an atom and λ its mean free path defined in such a way that an atom moving a distance λ will, on an average, lose completely its momentum parallel to a fixed direction. In applications to the present problem, λ will signify the mean free path, defined in the foregoing manner, for an atom threading its way through a mixture of two gases. By similar considerations we obtain

$$\beta_1=n_1m_1c_1/\lambda_{12}, \tag{15}$$

[1] Cf. N. Bohr, *Studier over Metallernes Elektronteori*, pp. 20 and 21, Copenhagen, 1911.

where λ_{12} denotes the mean free path of an atom of the first kind relative to atoms of the second kind only, but otherwise it is defined in the same manner as λ.

In a gas consisting of free electrons and positive ions these considerations do not give the total friction. In fact, due to the slow decrease of the force with increasing distance between two particles as calculated from the inverse square law, it is not certain that the kinetic theory of the gas mixture can be founded on the hypothesis of random distribution of electrons relative to atoms, as is tacitly done above. Broadly speaking, the conditions may be described as follows: On following an individual particle in its Brownian movement through the gas we shall find that it constantly attracts particles of the opposite sign and repels particles of the same sign, as a consequence of which it will, on an average, be attended by a small atmosphere of particles of the opposite sign. This atmosphere will only in exceptional cases be spherically symmetrical, and it will, therefore, in general exert an attraction on the specified particle, thus contributing to the internal viscosity in a manner not covered by the two terms discussed above. The additional terms introduced in this way will, as regards the form, be expected to duplicate Γ^e_{x1} and Γ^e_{x2} plus a hydrostatic term, but the values of the coefficients involved may be essentially different from those given above. The analytical discussion of this electrical viscosity is given in the appendix, and it appears that its effect is negligible for the problem at hand, as will be shown presently.

In the current applications of statistical principles to electron theory, it is customary to neglect all frictional terms except the one given by (13), and a discussion of the equations of motion (11) leads then in a very simple manner to the ordinary results of the classical electron theory of metals as regards electric conductivity and thermoelectric phenomena. Thus in the presence of an electric field, electrons and positive ions will be set moving in opposite directions, and this case is taken care of by the presence of the term in (11) which gives the external force on the particles. By using (15), the expression for the electric conductivity of a metallic conductor given by Lorentz follows immediately, apart from a numerical factor of the order unity. For the problem in hand, a further consideration

of this case is of little interest, since an external field of force which will keep electrons running continually in a closed circuit in the sun against the resistance offered by the atoms is unknown to us.

It will further be seen from (11) that an electric current may be produced by variations in pressure, a fact which is shown experimentally by the thermoelectric phenomena. Assume for a moment that an insulated metallic rod, as regards conduction of electricity, can be considered as consisting of fixed positive atoms between which electrons are free to move, being hampered in their motion by collisions with the metallic atoms only. If the rod is at a uniform temperature T and no external forces are acting, the electron pressure $p_e = n_e kT$ is a constant throughout the rod, except quite close to the surface where special forces must act to keep the electrons from escaping from the rod. Special consideration of the surface conditions are, however, unnecessary for the present purpose. Assume now that one end of the rod is cooled while the temperature of the other end is kept constant. Due to the decrease in p_e at the cold end of the rod, electrons will begin to flow from the warmer to the colder regions, and a thermocurrent is brought into play which will tend to separate positive and negative charges, the former being assumed to be fixed. Finally, a state of equilibrium is attained in which the diffusion of the electrons enforced by the temperature gradient is exactly counterbalanced by diffusion in the opposite direction enforced by an electric potential gradient $\partial\psi/\partial x$, etc., which from (11) is seen to be given by

$$\partial\psi/\partial x = -\frac{1}{n_e e}\partial p_e/\partial x\,, \text{ etc.}\,, \tag{16}$$

at any point inside the rod that is not so close to the surface that the neglected surface conditions play an essential part.

It was once an idea of Hale's[1] that similar processes might take place in sun-spots. Hale assumed that the primary phenomenon in the formation of a spot was a very strong and local lowering of temperature, which in turn would be expected to generate a radial electric current according to the principles considered above. If the spot was also a local hydrodynamic vortex, the stream lines of this

[1] G. E. Hale, *Mt. Wilson Contr.*, No. 71, p. 69; *Astrophysical Journal*, **38**, 97, 1913.

current would, by hypothesis, be deformed into spirals, and the transverse component of the current thus produced would then give rise to an axially symmetrical magnetic field.

There can scarcely be any doubt that thermoelectric phenomena of this kind must occur when a sun-spot is formed, and it may be shown that the difference between a gas and a solid metal does not change the conditions seriously. The essential difficulty in this theory seems to lie in the hypothesis that a radial current of the foregoing type would acquire a transverse component by rotation of the gases. In fact, superposing a rotation on the radial motion of the gases does not introduce any agency causing relative motion of electrons and atoms in a transverse direction, as the pressure gradient must be directed radially, irrespective of the spiral motion of the gases. The underlying conditions are, perhaps, best elucidated from the point of view of the circulation theorem as formulated by Bjerknes.[1] Suppose a certain closed curve to be drawn in the medium and to move as if it consisted of molecules belonging to the medium. The circulation, C, of such a curve is defined as the instantaneous line integral of the velocity of the medium taken around the given curve.

$$C=\int(udx+vdy+wdz)\,. \tag{17}$$

By forming the circulation of free electrons, C_e, and of positive atoms, C_a, separately, for a circular path with the spot as a center and situated in a plane perpendicular to the axis of the spot, it is clear that $n_e e(C_e-C_a)$, divided by the length of the curve, will be proportional to the current density at the requisite distance from the center of the spot. The circulation theorem now states that the variation in C per unit time, which is due to the particular distribution of pressure and density present, is, in ordinary vector notation, given by

$$dC/dt=\int\int[\nabla\alpha\nabla p]_n d\sigma\,, \tag{18}$$

the integral being extended to any surface bounded by the curve in question, α being the specific volume. If the vortex possesses circular

[1] Cf. *Dynamics of the Circular Vortex*, p. 9; *Geofysiske Publikationer*, **5**, No. 4, Oslo, 1921.

symmetry, the curve will be situated completely on a surface of constant pressure as well as of constant density, and the foregoing integral will be identically zero. The distribution of pressure and density in the vortex will therefore have no influence on the circulation of such a curve, and hence it will have no tendency to impress a systematic horizontal motion on electrons and atoms, taken separately. On the other hand, the maximum circulation impressed, according to (18), will be for a curve in a vertical plane through the axis of the spot, thus leading to a congestion of electricity at the axis. The free charges produced in this way must very soon react on the current, just as in the example with the insulated rod, and a steady state must soon be reached in which the variation in electronic pressure is balanced by an electric field. All things considered, it does not seem likely that the magnetic fields in sun-spots originate by thermocurrents of the foregoing type, and we must look for some other cause, both for the problem of magnetism in spots, as well as for the general magnetic field, which latter under no circumstances can be related to phenomena of the foregoing sort.

Turning to the discussion of the electric currents produced by the forces of internal viscosity, it will be apparent that these forces are of such a kind as to set up currents without producing space charges (as was the case with thermo-currents), and therefore will last as long as the motion lasts that brings viscosity into play. To bring out this point clearly, consider the case of a straight circular vortex. This case will correspond to the motion of the sun as a whole, as well as to a symmetrical spot on the solar surface. The motion will be assumed to be quasi-stationary. Choosing the axis of rotation as a z-axis, and simplifying equation (1) according to the foregoing assumptions, we arrive at the following expression for the current density:

$$I=-\frac{n_e e}{\beta}\left\{\kappa\frac{\partial}{\partial r}\left(2w+r\frac{\partial w}{\partial r}\right)+r\frac{\partial\kappa}{\partial r}\frac{\partial w}{\partial r}+r\frac{\partial}{\partial z}\left(\kappa\frac{\partial w}{\partial z}\right)\right\}\ ;\quad r^2=x^2+y^2\,, \tag{19}$$

where we have put

$$\kappa=\kappa_e/n_e m_e-\kappa_a/n_a m_a\ ;\qquad \beta=\beta_e/n_e m_e+\beta_a/n_a m_a\,, \tag{20}$$

and where ω is the angular velocity of the vortex at the point r, z. When the vortex rotates like a rigid body, ω is constant, and by (19) I becomes zero, as it should be. It is essential to note that in the general case the direction of the current is not determined by the direction of rotation of the vortex, and that the vortex will, in the general case, consist of a system of annular current tubes, the sign of the current changing when passing from a specified tube to an adjacent one. Without detailed analysis it will be seen that the electric currents produced by internal viscosity are qualitatively of the type which appear to underlie the phenomena of solar magnetism. As regards the sun-spots, the hypothesis requires the existence of circulating motion of some kind around a spot, an assumption which seems to be compatible with observational results. To explain the existence of the general magnetic field in the same manner, it is necessary to postulate the existence of a universal wind system, extending over most of the surface or interior of the sun, since, as remarked above, the uniform rotation can produce no effect at all. The sun, however, does not rotate like a rigid body, as the angular velocity increases on passing from pole to equator, and the foregoing hypothesis would thus require the existence of a causal connection between the general magnetic field and the non-uniform rotation of the sun.

The foregoing hypothesis seems to work well qualitatively, but very serious objections against it appear when attempting to develop a quantitative theory on this basis. It is for this purpose sufficient to consider a straight and infinitely long vortex of uniform composition, taking κ and β as constants and ω to be of the form $G(r)z^2/z_0^2$, where z_0 is a certain constant with dimensions of a length. The transverse dimensions of the vortex will be assumed to be finite. On these assumptions the magnetic force inside the vortex will be a function of r only, which is zero at an infinite distance from the axis. Calculating the line integral of the magnetic force along a contour consisting of two coplanar straight lines perpendicular to and in the plane of the vortex axis, and two lines parallel to this axis, we find, on using (2), that the magnetic field-strength at a distance r from the axis is given by

$$H = 4\pi \int_r^\infty I dr\,, \tag{21}$$

which, by (19) and by our assumption concerning the form of ω, becomes

$$H=\gamma\left\{\left(2\omega_r+r\frac{\partial\omega_r}{\partial r}\right)+\frac{2}{z_0^2}\int_r^\infty G(r)r\,dr\right\}\;;\quad \gamma=4\pi n_e e\kappa/\beta\,. \tag{22}$$

The first term of this expression depends only on the state of the vortex at the point under consideration, while the last term depends on the constitution of the vortex in the entire region external to this point. Suppose it is possible to define properly a certain mean radius, R, of the vortex, and a certain mean value, G_r, of G by the equation

$$\int_r^\infty 2G(r)r\,dr=G_r(R^2-r^2)\,. \tag{23}$$

Equation (22) then transforms into

$$H=\gamma\{2\omega_r+r\partial\omega_r/\partial r+G_r(R^2-r^2)/z_0^2\}\,. \tag{24}$$

The constitution of solar matter enters the problem through the single coefficient γ. Since the electronic mass is very much smaller than the mass of an atom, the terms involving m_a may be neglected in the expression of γ. Thus for the case in which the effects of ionic forces are neglected and κ and β are assumed to be given by the expressions (14) and (15), we obtain, approximately,

$$\gamma_c=\tfrac{1}{3}en_e\lambda_e\lambda_{ea}\,. \tag{25}$$

For the case of the "electric" viscosity discussed in the appendix, the value of γ is, as regards order of magnitude,

$$\gamma_e=n_e e\left(\frac{1}{\kappa}\right)^2, \tag{26}$$

and the only essential difference between the two cases is seen to consist in replacing the mean free path by the mean thickness $1/\kappa$ of the electric layer around an ion.

An upper limit to the mean free paths λ_e and λ_{ea} may be obtained in the following way. Consider a region so far down in the photosphere that the atoms may be assumed to be all ionized, with N free electrons per atom. An electron moving with a velocity v and

colliding with such an atom, considered to be at rest, will be deflected through an angle θ which is given by

$$\tan \theta/2 = Ne^2/pmv^2 , \tag{27}$$

where p is the distance from the atom to the direction of the initial path of the electron. Consider all collisions for which $\theta > \pi/2$ and for which, accordingly, $p < p_m = Ne^2/mv^2$. The number of such collisions suffered per second by an electron of the type in question is $\pi P_m^2 n_a v$ corresponding to a mean free path

$$\lambda = m^2v^4/\pi n_a N^2 e^4 \tag{28}$$

approximately. In addition to collisions of the foregoing type, the electron will also suffer collisions in which $\theta < \pi/2$ and which, of course, will change the momentum of the electron to some extent. An upper limit to λ_{ea}, as well as to λ_e, is thus given by the above expression alone, provided it is averaged over all velocities. For the case of thermal equilibrium the mean value of $(mv^2)^2$ is $15k^2T^2$, and the upper limit to γ_e is accordingly

$$\gamma_e = \frac{n_e e}{3}\left(\frac{15k^2T^2}{\pi n_a N^2 e^4}\right)^2 . \tag{29}$$

Introducing the expression of κ (eq. 37 in appendix), we find

$$\gamma_e = kT/4\pi e(N+1) . \tag{30}$$

It appears that for all cases of interest for the present problem, γ_e must be very small compared to γ_c. In order that $\gamma_e = \gamma_c$ in the sun's photosphere ($T = 6000°$, $N \to 1$), the number n_a would have to be of the order 10^{22}, a figure much larger than can be admitted on spectroscopic grounds, as well as on the basis of the known value of the sun's mean density. It seems, therefore, safe to conclude that the electrical viscosity is immaterial to this problem.

It is now frequently assumed that the pressure in the sun's reversing layer is as small as 10^{-4} atmosphere, which for $T = 6000°$, $N = 1$, would correspond to $n_a = 10^{13}$. The corresponding value of γ_c is about 10^6 c.g.s., and a sensibly larger value is hardly permissible.

The terms in the parenthesis of (24) have dimensions of angular velocities, and, in order of magnitude, neither of them can be expected to be greatly in excess of the angular velocity of the vortex itself at the point in question. Remembering that the magnetic field in sun-spots often attains values of several thousand gausses, and adopting the above value of γ_e, we find that this theory would require a spot to revolve with an angular velocity of several million revolutions per second. This figure, which in itself seems prohibitive enough, is only a lower limit. It is easy to convince one's self that the case is exactly the same for the general magnetic field.

This result would seem to require careful consideration, for the foregoing discussion apparently covers all available possibilities. In fact, the deductions have been based partly on the ordinary electron theory—including the validity of Coulomb's law as applied to collisions between electrified particles—and partly on the assumption that the free electrons considered are in temperature equilibrium with the solar substance. Coulomb's law may, for the collisions in question, be in error by minor amounts, but discrepancies such as would remove the difficulties are entirely out of question. Within the frame of ordinary electromagnetic theory there appears thus to be only a single possibility left for escape, viz., to assume that the electrons which are answerable for the magnetic fields are not in temperature equilibrium with the solar gases, and are capable of describing excessively long free paths; in short, that they are β-particles. Measurements of the mean range of homogeneous β-particles have been made by Varder.[1] In these experiments the velocities of the particles were about 0.85 of the velocity of light, and the mean range found corresponded roughly to the formula

$$\lambda = (2n_a m_a)^{-1} \text{ c.g.s.} \qquad (31)$$

For atomic weight 60 and $n_a = 10^{13}$, the value of λ is 5000 km. The range of such a particle is therefore enormous as compared with the mean free path of an ordinary electron, which under the same conditions and a temperature of 6000° would be of the order of 10 cm. Proceeding in a manner analogous to the derivation of the coefficient of

[1] *Philosophical Magazine*, **29**, 725, 1915.

viscosity in gas theory, we find that the electric current density due to emission of β-particles in a moving medium is given by an expression which is practically identical with that used for the conduction by free electrons, (19), provided the average range of a β-particle is substituted for the mean free path of an electron, and, further, that the problem is still characterized by a constant γ. Denoting the number of β-particles emitted from the solar substance per unit time and volume by Qn_a, and remembering that the mean time required by a particle with a velocity v to complete a range λ is λ/v, we obtain $n_e=Qn_a\lambda/v$, and, finally, as regards order of magnitude,

$$\gamma_c=\lambda^3 Qn_a e/3v\,. \tag{32}$$

By combination of (32), (31), and (24) it is now possible to estimate the upper limit Q^{-1} of the mean life-period of the hypothetical element in question. We arrive in this way at the result that Q^{-1} cannot possibly be longer than a few years. This estimate might be increased, provided the particles in question were emitted with larger initial velocities than assumed above. On the other hand, the theory involves several unwarranted approximations which, when removed, would reduce the foregoing limit very seriously. The most important of these approximations is perhaps the implicit assumption that the particles travel in straight lines. In fact, due to the action of the magnetic field, the path of such a particle will be very strongly curved, and in just such a way as to reduce the intensity of the field. The effective range to be introduced in (32) is therefore considerably smaller than the experimental value used above. In short, it does not seem likely that the foregoing estimate of Q^{-1} can be appreciably enlarged.[1] The conclusion follows that Q^{-1} is of an entirely different order of magnitude from the life-period of a star, and that therefore the radioactivity in question, if existing, must differ essentially in nature from radioactivity as it manifests itself in terrestrial experiments. When we consider the balance of energy in a star, difficulties still more serious arise. In fact, the

[1] Note added in proof. In expression (32) the secondary ionization produced by the primary particles is neglected. This ionization, however, is considerable, each primary particle producing some millions of ions. The ranges of secondary particles are known to be relatively small, and it does not seem probable that a correction in this sense will essentially alter the conclusions in the text.

energy of a fast-moving β-particle is enormous as compared with the energy residing in a quantum of radiation of visible light; and, relatively speaking, a very small number of such particles emitted per second, so far within the sun as to be absorbed, would be sufficient to account for the total solar radiation. As far as may be judged from the foregoing simple theory, it seems that the emission of β-particles of median velocities necessary to explain the general magnetic field would require a much larger emission of energy than is compatible with the known luminosity of the sun.

A way out of this difficulty, on the foregoing lines, is only attainable by assuming the β-particles in question to be very much more penetrating than any rays observed in terrestrial experiments, as theory would indicate that the range of such particles is proportional to the energy of expulsion,[1] while the magnetic field-strength, according to the above theory, would be proportional to the square of the range. By increasing the range indefinitely and keeping the magnetic field-strength constant, it is thus possible to reduce the total energy emitted below any assigned limit.

A further enlargement of the question does not seem profitable at present. We have discussed in this paper the general problem of accounting for solar magnetism on the basis of ordinary physical theory. Although it is possible to construct a theory which qualitatively meets observational requirements, the quantitative discussion does not lead to conclusions which stand in any immediate relation to observational results. The question whether the sun can be assumed to emit extremely penetrating corpuscular radiation should be carefully looked into. On the other hand, we have no a priori reason for expecting ordinary magnetic theory to apply to cosmical phenomena in all details. Whether one view or the other is taken, it seems rather certain that solar magnetism cannot be accounted for on the basis of physical facts with which we are completely familiar from ordinary terrestrial physics.

APPENDIX

Electrical stresses in an ionized gas.—Since electrical viscosity is due to the systematic distribution of particles of opposite sign in a

[1] Cf. N. Bohr, *Philosophical Magazine*, 30, 594, 1915.

moving gas, we shall follow, in deriving a first-order expression for the coefficient, a method first used by Debye and Hückel in the theory of electrolytes.[1] Consider an ionized gas consisting of free electrons and positive ions of r different kinds, the charge on an ion of the ith kind being $N_i e$, where $-e$ is the charge on an electron. The mean value of the electric potential at a point x, y, z, relative to a co-ordinate system which is fixed with respect to a particular ion, we denote by ψ and assume that it is, to a first approximation, given by Poisson's equation

$$\nabla^2\psi = -4\pi e \sum_1^r n_i N_i \,, \tag{33}$$

where n_i is the mean number of ions of the ith kind per unit volume at the point x, y, z, and where the free electrons are regarded as a particular kind of ions. In a state of equilibrium, the numbers n_i will be determined by the competing action of the electric forces, the motion of the gas, and ordinary spontaneous diffusion. Let V be the mean value of the vectorial velocity of all particles in the vicinity of the point in question. The mean velocity of each specified particle will differ from V by small amounts. Thus spontaneous diffusion introduces an additional mass motion with a velocity

$$V_d^i = -\nabla n_i D_i / n_i \,,$$

relative to the other gases, where D_i is the coefficient of diffusion of the ions of the ith kind. The electric forces will, on the other hand, induce a forced diffusion with a mean velocity

$$V_f^i = -\alpha_i N_i e \nabla\psi \,,$$

where α_i is the mobility of the i-ions. The total velocity of these ions is thus

$$V^i = V + V_d^i + V_f^i \,.$$

By the equation of continuity,

$$\partial(n_i V^i)/\partial t + \text{div}\,(n_i V^i) = 0 \,,$$

[1] Cf. *Physikalische Zeitschrift*, **24**, 1, 1923.

we have that n_i, under stationary conditions, is determined by the equation

$$\text{div}\,(-D_i\nabla n_i - a_i N_i e n_i \nabla\psi + n_i V) = 0\,. \tag{34}$$

If V is zero, this equation has the solution

$$n_i = n_i^0 e^{-a_i N_i e\psi/D_i}\,, \tag{35}$$

where n_i^0 is the value of n_i at a point where ψ is zero. Since this expression must coincide with the Boltzmann-Maxwell distribution law, it is necessary, as was first pointed out by Einstein, that

$$D_i/a_i = kT\,, \tag{36}$$

where k is Boltzmann's constant and T the absolute temperature.

We wish ultimately to determine ψ; and three simple cases will be considered.

1. $V=0$.—The n's are then given by (35). Introducing these expressions in (33) and expanding the exponentials to the first order, we obtain

$$\nabla^2\psi = \kappa^2\psi\,; \qquad \kappa^2 = 4\pi e^2 \Sigma n_i^0 N_i^2/kT \tag{37}$$

the solution of which, for an electric point charge $N_i e$ at the origin, is

$$\psi = N_i e\, e^{-\kappa r}\,; \qquad r^2 = x^2+y^2+z^2\,. \tag{38}$$

2. $V = const.$—The specified ion is then moving relative to the gas with a velocity $-V$. By (33) and (34)

$$\nabla^4\psi - \left(\frac{4\pi e^2}{kT}\Sigma n_i N_i\right)\nabla^2\psi = -4\pi e\Sigma\frac{N_i}{D_i}(\nabla n_i V) + \frac{4\pi e^2}{kT}\Sigma N_i(\nabla n_i \nabla\psi)\,. \tag{39}$$

To reduce this equation to a first approximation, we must assume $n_i = n_i^0 =$ constant in the left-hand term and neglect the last term on the right. The term in V we consider as a small correction and therefore we suppose n_i here to be given by (35) expanded to the first order, and the value of ψ involved to be given by (38). By

taking the direction of V as the axis of x, equation (39) reduces to the final form

$$\nabla^2(\nabla^2-\kappa^2)\psi=N_iV\sigma\frac{x}{r}\frac{\partial}{\partial r}(e^{-\kappa r}/r)\,; \qquad \sigma=\frac{4\pi e^2}{kT}\Sigma n_i^0N_i^2/D_i\,. \tag{40}$$

This equation has the intermediate solution

$$\nabla^2\psi=\kappa^2N_ie\ e^{-\kappa r}/r+N_iV\sigma\times e^{-\kappa r}/2r\,, \tag{41}$$

which gives finally

$$\psi=N_ie\ e^{-\kappa r}/r+\{1-2e^{\kappa r}/\kappa^2r^2-2e^{\kappa r}\frac{d}{dr}(e^{-\kappa r}/r\kappa^2)\}N_iV\sigma\times e^{-\kappa r}/2r\kappa^2\,. \tag{42}$$

3. $V_x=$*linear function of* z; $V_y=V_z=0$.—We introduce a new co-ordinate system with axes parallel to the original axes, but moving with a velocity equal to the velocity of the fluid at the point in question. This problem leads to an equation of the form (40). All we need for the present is an intermediate solution corresponding to (41), and this is found to be

$$\nabla^2\psi=\kappa^2N_ie\ e^{-\kappa r}/r+N_i\frac{\partial V_x}{\partial z}\frac{\sigma x}{4}e^{-\kappa r}/r\,, \tag{43}$$

where $\partial V/\partial z$ has been regarded as a constant.

We are now prepared for the discussion of the electrical stresses in the moving gas. Consider the momentum parallel to the x-axis transferred per unit area by electrical forces from the upper to the lower side of a plane parallel to the xy-plane. We specify a certain particle, fixed at a distance ζ below the given plane. The particle will be attracted or repelled by the particles above the plane with a mean resultant force the x-component of which we shall denote by f_x^i and calculate by means of expression (43). Introducing a co-ordinate system with axes parallel to the original ones and with origin in the particle, we find for f_x^i the expression

$$f_x^i=N_ie/4\pi\int\frac{\nabla^2\psi x}{r^3}dxdydz\,, \tag{44}$$

where the integration is to be extended to the whole gas above the given plane. The value of $\nabla^2\psi$, however, will decrease so rapidly with

increasing distance from the particle that only a very thin layer of gas just above the plane will contribute appreciably to the integral in question. The first term on the right-hand side of (43) is symmetrical with respect to positive and negative directions of x, and will therefore contribute nothing to the x-component of the force. The second term, on the other hand, corresponds to a distribution of electricity of opposite sign for the two directions, and will therefore in general determine a value of f_x^i different from zero. Performing the integration, we obtain

$$f_x^i = \pi N_i^2 e\sigma\phi(x\zeta)\frac{\partial V_x}{\partial z}/16\kappa^2\,; \quad \phi(x) = \frac{1}{\pi}\int_x^\infty e^{-u}(1-x^2/u^2)^2\,u\,du. \qquad (45)$$

Multiplying f_x^i by n_i^0 and integrating from $\zeta = 0$ to $\zeta = \infty$, we obtain as an expression for the total momentum parallel to the x-axis transferred per unit time and area through the plane in question, and to particles of the ith kind only,

$$-\Gamma_{xz}^i = a\pi e\sigma n_i^0 N_i^2 \frac{\partial V_x}{\partial z}/8\kappa^3\,; \qquad a = \int_0^\infty \phi(x)dx\,, \qquad (46)$$

where a is a pure number of order unity. The coefficient of electric viscosity is, therefore, as regards first-order quantities, given by

$$\kappa_i^e = n_i^0 N_i^2 a\pi e\sigma/8\kappa^3\,. \qquad (47)$$

The additional resistance experienced by an electric particle moving with a velocity V through an ionized gas is simply obtained from (42). The first term in ψ is symmetrical and will give rise to no forces on the particle. Expanding the second term to first-order quantities, close to the origin, we find that the particle is acted upon by a force which is directed opposite to the velocity and equal to

$$\beta_i^e V = 4\kappa^2\kappa_i^e V/\pi a\,. \qquad (48)$$

The hydrostatic electric pressure might be calculated in an analogous manner, but has no interest for the present purpose.[1]

MOUNT WILSON OBSERVATORY
July 1925

[1] *Monthly Notices*, **84**, 723, 1924.

PLATE XIV

N

E

W

N.G.C. 6822

Photographed at the 42-foot focus of the 100-inch Hooker telescope on July 10, 1923. Exposure 3^h30^m on a Seed 30 plate. Enlargement 2.5 times original negative.

N.G.C. 6822, A REMOTE STELLAR SYSTEM[1]

By EDWIN HUBBLE

ABSTRACT

A study of N.G.C. 6822.—The object is a very faint cluster of stars and nebulae, about $20' \times 10'$, resembling the Magellanic Clouds. Detailed investigations confirm the similarity and show that it extends to many structural features of these systems. Familiar relations such as those connecting periods and luminosities of Cepheids, luminosities of brightest stars involved in diffuse nebulae, and frequencies of the most luminous stars in the systems are consistent when applied to N.G.C. 6822, the first object definitely assigned to a region outside the galactic system.

Variable stars.—Eleven of the fifteen variables found in the cluster are Cepheids. A system of photographic magnitudes has been established from comparisons with the two nearest Selected Areas; and periods, light-curves, and magnitudes have been determined for the Cepheids. The periods range from 12 to 64 days and the magnitudes at maximum, from 17.45 to 19.05. The period-luminosity relation is conspicuous, and a comparison with Shapley's general curve in *Harvard Circular*, No. 280, indicates a distance for N.G.C. 6822 of 214,000 parsecs. $m - M = 21.65$.

Nebulae.—Five diffuse nebulae are involved in the cluster. The four brightest show emission spectra similar to those of the diffuse galactic nebulae. A radial velocity of +25 km/sec., uncorrected for solar motion, has been derived from one slit spectrogram of the brightest nebula. The mean diameter of the diffuse nebulae, about 40 parsecs, is comparable with those of the largest nebulae in other systems. In addition to the diffuse nebulae, several small objects are present which are probably non-galactic nebulae whose images are superposed on that of the cluster.

Star counts.—The distribution of apparent photographic magnitudes has been determined down to 19.4, corresponding to absolute magnitude −2.25. The brightest stars in the system are of absolute magnitude about −5.8. When a mean color-index of +1.35 is assigned to the cluster stars, their frequencies are very similar to those of the absolute visual magnitudes in the vicinity of the sun.

Luminosity and space density.—The surface brightness of the central core is about 22.1 photographic magnitudes per square second of arc. This value leads to a total absolute magnitude of −12.0 for the core, or −12.7 for the entire system, and to a space density of about 6.1 absolute magnitudes per cubic parsec for the core, or 8.8 for the entire system.

Cepheid criterion of distance.—Since the Cepheid criterion appears to function normally in N.G.C. 6822, its value as a means of exploring extragalactic space is considerably enhanced.

Influence of instruments on observations of nebulae.—A summary of the early observations of N.G.C. 6822 illustrates the confusion which may arise from disregarding the limitations of various types of instrument and emphasizes the caution required in discussing observations of nebulae.

N.G.C. 6822 is a faint irregular cluster of stars with several small nebulae involved. The position for 1925 is $\alpha = 19^h\,40.7^m$, $\delta = -15°$, galactic latitude $-20°$. The general appearance is shown on Plate XIV, enlarged from an exposure of three hours and a half with the 100-inch reflector. The total area covered by the cluster is about $20' \times 10'$, but the most conspicuous feature is a core about $8' \times 3'$ in

[1] *Contributions from the Mount Wilson Observatory*, No. 304.

which the star images are densely crowded. The photographs show a striking resemblance to the Magellanic Clouds, and detailed investigations have abundantly confirmed this first impression. Cepheid variables, diffuse nebulae, dimensions, density, and distribution of stellar luminosities agree in defining the system as a curiously faithful copy of the clouds, but removed to a vastly greater distance. N.G.C. 6822 lies far outside the limits of the galactic system, even as outlined by the globular clusters, and hence may serve as a stepping-stone for speculations concerning the habitants of space beyond.

EARLY OBSERVATIONS

The early observations are confused, as is usual for faint complex objects. In the case of N.G.C. 6822 the confusion is clearly the result of disregarding the limitations of the various instruments employed. A short history will justify this statement and emphasize the caution required in discussing observations of nebulae. Barnard discovered[1] the object, apparently with his own 5-inch refractor. He described it as very faint and stated that some time later, when he determined its position with a 6-inch equatorial, it was seen with difficulty. The next year, however, he examined it with the 6-inch and a comet eyepiece. It was fairly conspicuous, large and round, about 10′ or 15′ in diameter, rather dense and even in its light. Barnard remarked:

> It certainly seems much larger and much denser than last year and I certainly think it has increased in density and size since that time. . . . . If it had always been as large and bright as now, I cannot conceive how it could have been missed by observers when examining G.C. 4510 (the bright planetary N.G.C. 6818, which is about 40′ north preceding). Probably this is a variable nebula.[2]

The comet eyepiece accounts for these discrepancies. The 5-inch, it seems, was not equipped with this accessory, nor would one be used in determining positions with the 6-inch. So large and faint a cluster is best seen with a low power and a wide field. It will not stand magnification. At Mount Wilson, for instance, N.G.C. 6822 is fairly conspicuous in a short 4-inch finder with a low-power eyepiece, but is barely discernible at the primary focus of the 100-

[1] *The Sidereal Messenger*, **3**, 254, 1884. [2] *Ibid.*, **5**, 31, 1886.

inch. The latter, however, shows the small bright details which are invisible in the finder. This factor is apparent in the later observations.

Barnard's remarks obviously refer to the cluster as a whole and furnish the description in the *N.G.C.*—"vF, L, E, dif." During the next year, 1887, the region was examined[1] with the 26-inch refractor at the Leander McCormick Observatory. The relatively high magnification and restricted field conspired to render the cluster inconspicuous. It was missed entirely by the observer. The two brightest of the small involved nebulae, however, were seen and measured. Curiously enough, they were both attributed to Barnard and the discrepancy with the *N.G.C.* description was overlooked. When the *First Index Catalogue* was compiled, the brighter of these two objects was assumed to be N.G.C. 6822, although the description in the *N.G.C.* was not amended, and the fainter object was listed as a new nebula, "I.C. 1308, eF, eS, lE, gbM, 6822 p 12s." H. A. Howe,[2] with a 20-inch refractor, found only the brighter of these two small nebulae and remarked that N.G.C. 6822 was "very small" instead of "large" as described by Barnard. Bigourdan[3] with a 12-inch could see neither of the small nebulae, nor, of course, the cluster. His instrument was in the blind zone, too large for the cluster and too small for the involved nebulae.

The first photographs were made in 1906 and 1907 by Wolf[4] at Heidelberg with the 16-inch Bruce camera and the 28.5-inch Walz reflector. He identified the two small nebulae as N.G.C. 6822 and I.C. 1308, and announced the cluster, Barnard's original discovery, as a new object which was duly listed in the *Second Index Catalogue* as "I.C. 4895, group of neb., 25′ diam." Wolf described the cluster as an exceptionally dense *Nebelfleckhaufen*, or cluster of small nebulae, similar to those more open clusters to whose study he has contributed so largely. This remarkable interpretation existed for fifteen years unchallenged and apparently unnoticed.

Finally in 1922, Perrine[5] published a fair description from a plate made with the 30-inch reflector at Córdoba. He properly identified

[1] *Publications of the Leander McCormick Observatory*, **1**, Part 6.

[2] *Monthly Notices*, **60**, 137, 1900.

[3] *Observations des Nebuleuses*, **5**.

[4] *Astronomische Nachrichten*, **176**, 109, 1907.

[5] *Monthly Notices*, **82**, 489, 1922.

the entire system as Barnard's nebula, N.G.C. 6822, and described it as a cluster of stars and nebulosity resembling the Magellanic Clouds.

Meanwhile, the object had been photographed at Mount Wilson with the 10-inch camera, and the anomalous character of its image[1] had led to its being placed on the program for further study with the large reflectors. In July, 1921, Duncan[1] obtained two plates with the 100-inch reflector, on which the resemblance to the Magellanic Clouds was very striking. Detailed investigation was undertaken in July, 1923. The object was followed carefully through the two seasons of 1923 and 1924, and some plates are available for the seasons of 1921 and 1925. More than fifty direct exposures and several spectrograms have been made with the large reflectors.

CEPHEID VARIABLES

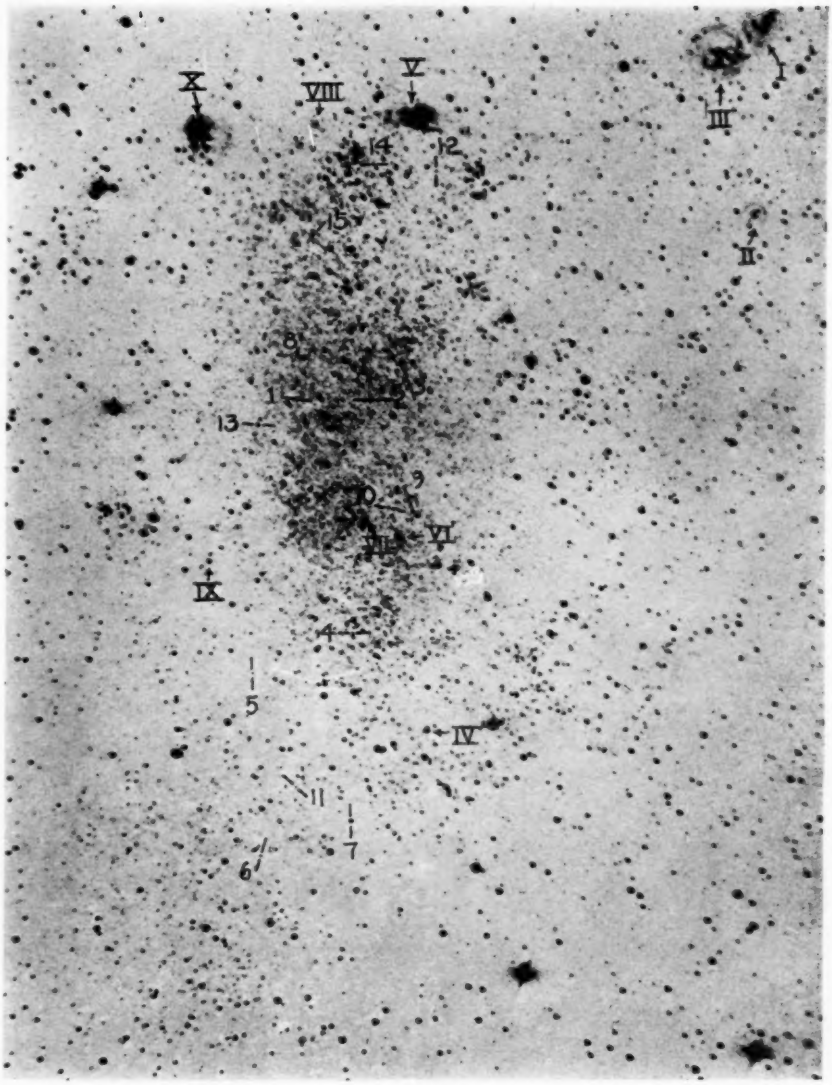
A survey of the plates with a blink-comparator has led to the discovery of fifteen variable stars within the area covered by the system. These are marked on Plate XV, and are identified by Arabic numerals. No variables have been found on the plates outside the limits of the cluster, hence those within the limits must be considered as actual members of the system. Eleven of the variables are clearly Cepheids, and a few of the remaining four may ultimately prove to be of the same type. Periods, normal curves, and photographic magnitudes have been determined for the Cepheids, the periods ranging from 12 to 64 days and the magnitudes, at maximum, from 17.45 to 19.05. The familiar period-luminosity relation is conspicuously present and the curve is parallel to Shapley's general curve expressing the corresponding relation among previously known Cepheids. The displacement in magnitudes between these two curves leads to a value of $m-M=21.65$, corresponding to a distance of 214,000 parsecs.

The system of magnitudes is based on nine Seed 30 plates taken with the 100-inch reflector. The zero-point and scale were determined from four 30-minute comparisons with the neighboring Selected Area 136 and two 20-minute comparisons with Selected

[1] *Mt. Wilson Contr.*, No. 256; *Astrophysical Journal*, 57, 137, 1923. The plates are mentioned earlier in the *Mt. Wilson Annual Report for 1921*.

PLATE XV

N



N.G.C. 6822

Negative print of Plate XIV. Variable stars are designated by Arabic figures; nebulae involved in or superposed on the cluster by Roman numerals.

Area 135. The plates were centered on three regions of the cluster —the center and points 3′ north and south, respectively—in order to reduce the effect of the distance correction. These comparisons gave magnitudes comparable with those of the Selected Areas down to the limits of the Mount Wilson measures[1] of the latter, about 18.0, and by a legitimate extrapolation, to a fraction of a magnitude fainter. The system was extended by means of three longer exposures, two of one hour each, and one of three hours and a half.

The internal agreement of the measures is very satisfactory down to about 19.1. Average deviations of individual plates from the mean are less than 0.1 mag. and the maximum range for an individual star on all the plates is 0.4 mag. Extrapolations beyond 18.0 depend upon the reliability of the faintest magnitude and a half of the scales in the two Selected Areas. If these are free from systematic errors, as is hoped, the extrapolation can be trusted for at least a magnitude. Distance corrections have never been thoroughly investigated for the field of the 100-inch, but the present measures were confined to the immediate vicinity of the optical axis, and approximate values of the corrections were derived from the measures themselves.

There is, however, a troublesome source of systematic error which has not been eliminated. Under ordinary observing conditions, long exposures at low altitudes with the 100-inch give images of appreciable dimensions. This is due to atmospheric refraction and changing seeing, focus, and figure. The result is that the faintest images are irregular and vary in density rather than in size. Above a certain limiting magnitude the effect is inappreciable, but as measures are carried down to fainter stars, the photometry of approximately point sources merges into photometry of surfaces. When the measuring is done with an ordinary scale-plate, the resulting magnitudes for the fainter stars become increasingly too bright. In the present measures, this effect becomes noticeable beyond about 19.1. Magnitudes fainter than this are increasingly unreliable, while those brighter are probably correct within 0.2.

A series of comparison stars was established for each variable in

[1] These measures on the international scale are unpublished but were available through the kindness of Mr. Seares.

TABLE I

PHOTOGRAPHIC MAGNITUDES OF VARIABLE STARS IN N.G.C. 6822

J.D.†	Q	CEPHEID VARIABLES											OTHER VARIABLES			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2875.44.....	P	18.85	18.5	.....	.....	18.85	.....	18.2	.....	18.8	.....	.....	18.65	18.1	18.5	18.85
2876.46.....	G	18.9	18.6	19.0	18.95	18.95	19.1	18.15	18.8	18.8	18.95	19.15	18.65	18.2	18.5	18.75
3611.50.....	E	18.2	18.3	19.0	18.35	18.9	18.75	17.45	18.85	19.2	19.3	19.2	18.85	18.2	18.75	18.75
3616.50.....	G	18.1	18.4	18.3	18.75	19.05	19.05	17.55	18.85	19.1	19.2	19.15	18.8	18.1	18.8	18.85
3619.53.....	E	18.4	18.6	18.45	19.0	19.15	19.25	17.55	18.95	18.85	19.25	19.3	19.05	18.2	18.8	18.7
3619.55.....	G	18.4	18.6	18.55	18.95	19.15	19.2*	17.65	18.95	18.85	19.2	19.3	19.0	18.25	18.75	18.65
3638.37.....	P	18.85	18.0	18.95	19.0	18.95	19.1	18.0	.....	.....	.....	.....	18.8	17.9	18.85	.....
3638.51.....	F	18.75	18.0	19.05	19.0	19.05	19.1	18.0	19.0	.....	19.1	19.0	18.85	17.95	18.75	18.85
3639.40.....	F	18.85	17.9	19.0*	19.15	18.9	19.2	18.05	18.85	18.9	19.2	19.05	18.75	17.95	18.7	18.85
3640.48.....	P	18.6	18.0	18.9*	.....	19.0	.....	18.05	.....	.....	.....	19.15	18.7	17.9	18.8	18.85
3641.50.....	E	18.55	18.05	19.0	18.95	18.9	19.15	18.15	18.6	19.1	19.2	19.2	18.85	17.85	18.8	18.85
3647.35.....	F	18.05	18.1	18.4	18.6	19.15	18.45	18.1	19.0	19.1	19.3	19.2	18.7	17.75	18.9	18.7
3678.38.....	G	18.1	17.85	18.45	18.95	19.05	19.15	17.5	18.95	19.3	19.2	19.1	18.75	17.9	18.7	18.75
3697.31.....	F	19.0	18.6	19.0*	18.7	19.05	19.1	18.05	19.0	19.1	19.2*	19.3	18.75	18.1	18.7	18.75
3698.33.....	G	19.05	18.65	19.05	18.45	18.95	19.2	17.9	18.95	19.1	19.3	19.0	18.8	18.05	18.75	18.7
3969.52.....	G	19.0	18.9	18.5	18.9*	19.0	18.75	18.1	18.8	19.2	19.0	19.15	18.2	18.1	18.7	18.45
3971.51.....	F	18.9	18.8	18.5	18.9	18.9	18.9	18.15	18.7	18.75	18.85	19.15	18.3	18.1	18.8	18.4
3972.51.....	P	18.9	.....	.....	.....	19.0	19.1	18.1	.....	.....	.....	19.15	.....	18.2	18.75	18.4
3990.45.....	G	18.8	18.3	19.05	18.85	19.1	18.75	17.8	18.9	19.0	18.8	19.2	18.05	18.3	18.4	18.45
3994.45.....	F	18.9	18.4	18.35	18.6	19.1	19.05	17.55	18.65	19.2	19.1	19.05	18.15	18.1	18.7	18.4
3995.45.....	G	19.0	18.45	18.4	18.75	19.15	19.1	17.65	18.75	19.05	18.9	19.2	18.2	18.1	18.65	18.5
3996.40.....	E	18.9	18.7	18.4	18.8	19.15	19.2	17.5	18.65	18.95	19.0	19.3	18.1	18.1	18.5	18.5
3997.40.....	G	19.0	18.5	18.4	18.9	18.95	19.1	17.55	18.85	19.1	19.1	19.3	18.2	18.2	18.6	18.4
3998.44.....	E	18.9	18.6	18.4	18.85	18.8	19.2	17.55	18.85	19.3	18.95	19.25	18.1	18.1	18.6	18.45
4002.37.....	E	19.05	18.7	18.65	19.0	19.15	19.25	17.65	18.9	19.15	18.95	19.35	18.0	17.95	18.4	18.45
4003.37.....	G	19.0	18.9	18.8	18.9	19.15	19.25	17.65	18.85	19.0	18.9	19.05	18.1	17.9	18.6	18.4
4004.37.....	E	18.9	19.0	18.7	18.95	19.1	19.1	17.6	19.0	18.8	18.8	19.05	18.05	18.05	18.5	18.5

4024.34.....	P	19.05	18.25	18.4	18.9	19.0	19.2	17.9	19.0	18.9	18.85	.....	18.2	17.8	18.55	18.45
4025.34.....	F	19.0	18.3	18.45	18.8	18.95	19.1	18.0	19.0	18.8	18.8	19.15	18.25	17.75	18.5	18.5
4026.40.....	P	18.85	18.4	18.5	18.75	18.85	18.55	17.9	18.85	18.9	19.1	19.05	18.35	17.75	18.7	18.4
4031.33.....	E	19.0	18.7	18.8	18.85	19.15	18.8	18.05	18.85	19.3	18.9	19.25	18.25	17.8	18.5	18.4
4032.33.....	G	18.95	18.8	18.7	18.9	19.05	18.8	17.95	18.85	19.1	18.9	19.25	18.25	17.8	18.6	18.5
4049.35.....	P	18.45	18.2	18.9*	18.8	.....	18.55	18.1	.....	.....	.....	.....	18.4	17.95	18.5	18.5
4050.35.....	F	18.6	17.9	19.1	18.9	19.1	18.7	18.1	18.8	19.2	18.9	19.2	18.4	18.0	18.55	18.5
4054.32.....	P	18.65	18.1	18.4	19.1	18.75	19.05	17.95	18.8	19.0	19.0	19.15	18.2	17.95	18.55	18.55
4056.32.....	G	18.75	18.15	18.3	18.95	18.95	19.0	17.75	18.6	18.85	18.8	19.2	18.2	17.9	18.6	18.4
4057.32.....	F	18.8	18.1	18.55	18.9	19.05	19.2	17.65	18.8	18.8	18.8	.....	18.3	17.9	18.6	18.5
4058.32.....	E	18.8	18.4	18.45	19.05	19.1	19.1	17.5	18.65	18.8	18.85	19.1§	18.35	18.0	18.55	18.45
4059.32.....	F	18.9	18.4	18.6	18.8	19.1	19.1*	17.5	18.8	18.85	19.0	.....	18.3	18.05	18.6	18.55
4085.30.....	P	19.0	18.1	18.3	18.85	19.0	19.2	17.85	18.8	19.1	18.8	19.3	18.3	18.3	18.5	18.5
4090.30.....	P	19.0	17.9	19.0	18.95	.....	18.6	17.9	.....	18.8	18.8	.....	18.2	18.25	18.5	18.6
4298.6.....	F	18.7	18.7	19.0	19.1	19.15	19.2	18.1	.....	19.2	19.3	.....	18.8	18.1	18.5	18.7
4319.6.....	G	18.3	18.0	18.4	18.9	18.85	19.15	17.55	19.0	19.3	.....	.....	18.9	18.2	18.6	18.8

* Limiting magnitude of the plate; variable invisible.
† Figures in this column are to be added to 2,420,000.0. The decimals represent Pacific mean time.
§ Discordant observation.

the manner described above, with successive steps averaging about 0.2 mag. These furnished easy and accurate means of following the variables. The data for the latter from the separate plates are collected in Table I. The first column gives the Julian Day repre-

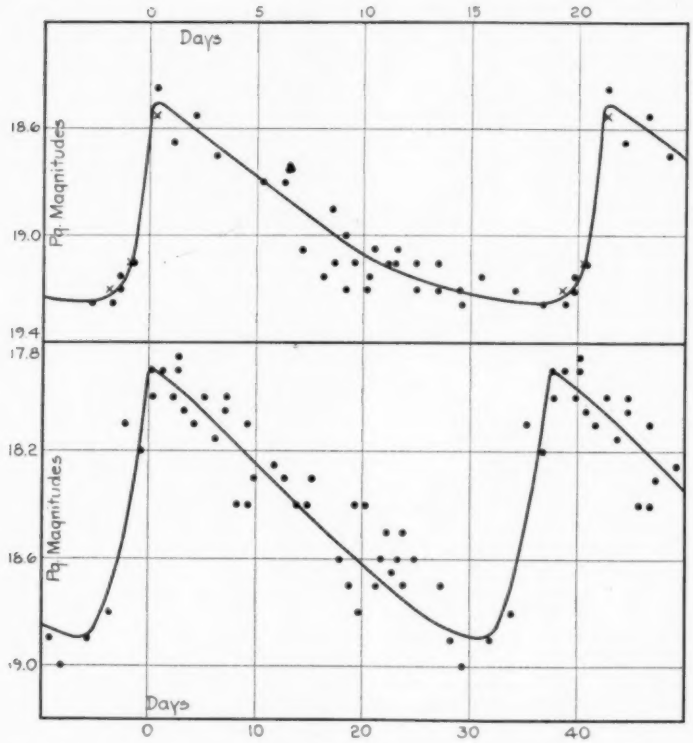


FIG. 1.—Light curves for two Cepheids in N.G.C. 6822. Upper curve, variable No. 6. Period 21.06 days; range 18.5–19.25. Lower curve, variable No. 2. Period 37.45 days; range 17.9–18.9. The three crosses on the rising slope of the upper curve represent observations on successive days and illustrate the rapid brightening of the variables.

senting Pacific mean time; the second, the quality of the plates, the letters "E," "G," "F," "P" and "vP" having their usual significance of "excellent," "good," "fair," "poor," and "very poor"; the remaining columns give the photographic magnitudes of the variables, which are designated by the numbers used on Plate XV. The exposures were usually from 60 to 75 minutes on Seed 30 plates.

The elements of the light-curves derived from these data for the eleven Cepheids are given in Table II. The small amplitudes found for the fainter stars are probably due, in part at least, to the systematic error previously mentioned. In order to emphasize their Cepheid characteristics two of the light-curves are illustrated in Figure 1. The other curves can easily be constructed from the data in Tables I and II.

The period-luminosity relation is shown in Figure 2, in which the logarithms of the periods in days are plotted against the photographic magnitudes at maxima. This departure from the customary

TABLE II

NORMAL CURVES FOR CEPHEIDS

Var. No.	Epoch J.D.	Period in Days	log *P*	Max.	Min.
7.....	2423994.0	64.0	1.806	17.45	18.15
2.....	4050.0	37.45	1.574	17.9	18.9
1.....	3678.0	30.55	1.485	18.1	19.0
3.....	3994.0	29.2	1.47	18.35	19.1
6.....	3647.0	21.06	1.325	18.5	19.25
8.....	3972.0	20.72	1.315	18.6	19.0
4.....	3992.0	17.31	1.238	18.5	19.1
9.....	3971.0	16.86	1.225	18.75	19.25
10*.....	.....	16.5±	1.22±	18.8	19.2
5.....	3699.0	13.87	1.142	18.8	19.2
11.....	4003.0	11.77	1.071	19.05	19.35

* Observations uncertain but Cepheid characteristics are pronounced. Rejection would not affect the period-luminosity curve appreciably.

Magnitudes fainter than 19.0 are probably systematically too bright, the effect increasing with the numerical value of the magnitudes.

usage of median magnitudes is required by the uncertainties in the fainter magnitudes, including most of those at minima. The full-line curve is the linear relation

$$m(\text{max.}) = 21.401 - 2.193 \log P ,$$

derived by a least-squares solution. The residuals are of the order of accuracy of the measures, averaging ±0.06 mag., with a maximum value of 0.20. This dispersion is surprisingly small and, in a measure, justifies the use of maximum rather than of median magnitudes. Furthermore, it excludes the possibility that the apparent magnitudes of the Cepheids might be sensibly affected by local absorption in the system.

The determination of the distance of N.G.C. 6822 is based on

Shapley's latest revision[1] of the photographic period-luminosity curve. The form of this curve depends on more than a hundred Cepheids in the small Magellanic Cloud and should be definitive except in the region of very long periods. The zero-point[2] was derived from galactic Cepheids of intermediate periods.

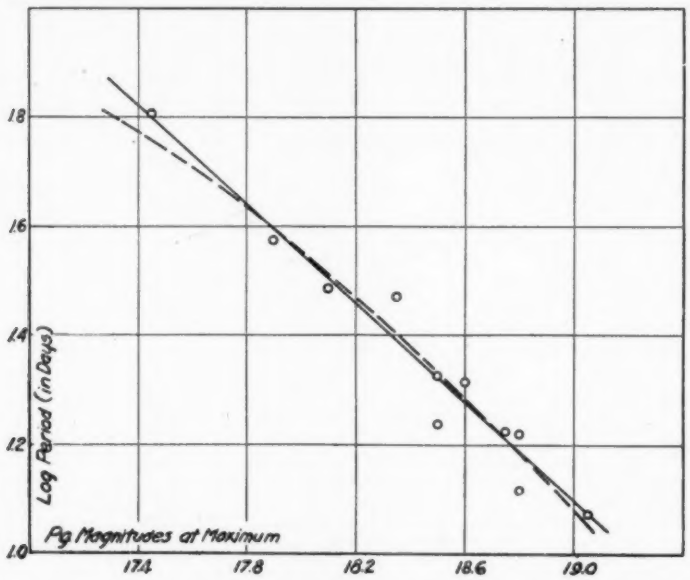


FIG. 2.—Period-luminosity relation among Cepheids in N.G.C. 6822. The continuous line represents the correlation curve, $m(\max) = 21.40 - 2.19 \log P$. The broken curve indicates the corresponding portion of Shapley's general period-luminosity curve reduced to the scale of photographic magnitudes.

In Figure 2, Shapley's curve is represented by the dotted-line curve adjusted to fit the data for N.G.C. 6822 by shifting it along the magnitude axis. The uncorrected modulus is

$$m\ (\text{max.}) - M\ (\text{median}) = 21.26 \pm 0.02.$$

The maximum residual is 0.20 mag., and the mean ±0.11 mag.

[1] *Harvard Circular*, No. 280.

[2] Recent results by Strömberg for the solar motion referred to the galactic Cepheids indicate the possibility of a substantial correction to the zero-point (*Mt. Wilson Contr.*, No. 293; *Astrophysical Journal*, **61**, 363, 1925). But until the question can be discussed in a definitive manner it seems best to use Shapley's value, with a full realization, however, of the possible necessity of a subsequent revision of all distances derived from the Cepheid criterion.

Since Shapley's curve is based on median magnitudes, the value of the modulus must be increased by half the mean range of the Cepheids in N.G.C. 6822. This quantity is somewhat uncertain, owing to the systematic error affecting the fainter magnitudes at minimum. The mean for all the Cepheids as listed in Table II is 0.67 mag., while for those brighter than 19.1 at minimum, the mean is 0.73 mag. Even this latter value appears to be low, judged by the apparent systematic decrease in range with decreasing luminosity and the ranges of Cepheids in other systems.[1] A value around 0.8 mag. would probably be nearer the truth. That actually used was 0.78 mag. The resulting values of the modulus and the distance are

$$m-M=21.65$$
$$\pi=0''.00000468$$
$$\text{Distance}=214{,}000 \text{ parsecs}$$
$$=700{,}000 \text{ light-years}$$

On the assumption of the universal validity of the period-luminosity relation, the principal source of uncertainty in this result arises from that affecting the zero-point of Shapley's general relation. In addition there is the uncertainty in the extrapolated magnitude scales for the Cepheids in N.G.C. 6822. This is difficult to estimate, but probably does not exceed 0.2 mag. The error arising from accidental sources is insignificant, probably less than 0.05 mag. in the modulus, or 2.5 per cent in the distance.

As for the general validity of the period-luminosity relation, this formula has hitherto led invariably to estimates of distances which are consistent with other characteristics of the systems to which it has been applied. In the case of N.G.C. 6822 such a relation certainly exists, and, within the limits of observational error, its form, as shown by Figure 1, is the same as that observed elsewhere. The only questionable feature is the zero-point, and even here

[1] A frequency curve for ranges of galactic Cepheids, based on data derived from Shapley (*Mt. Wilson Contr.*, No. 153; *Astrophysical Journal*, **48**, 279, 1918; and from the table compiled by the Committee on Variable Stars of the Astronomische Gesellschaft, *Vierteljahrsschrift der A. G.*, **59**, 298, 1924), shows a well-marked maximum at 0.8 mag. The data, however, are not homogeneous and, for the most part, are visual. Moreover, the relation between period and range has never been satisfactorily investigated.

corroborative evidence is not lacking. This is presented in detail later, but reference may be made at this point to Shapley's estimate of the distance of N.G.C. 6822.

He boldly assumed an analogy with the Magellanic Clouds, and, comparing angular dimensions, size, and luminosity of the diffuse nebulae, and estimated magnitudes of the brightest stars involved, arrived at a distance "on the order of a million light-years." This figure is comparable with that derived from the Cepheids and represents a brilliant application of the general principle of the uniformity of nature.

VARIABLES OTHER THAN CEPHEIDS

For completeness, the observational data for the four other variables found in the system have been included in Table I. Concerning three of them, nothing is known beyond the fact that they vary through small ranges and apparently in an irregular manner. The remaining variable (No. 12) ranges from about 18.1 to 18.9 or more. In July, 1922, it was moderately faint; in 1923, July to October, it was faint and slowly rising; in 1924, July to November, it was bright and may have passed through a maximum about the beginning of August; in 1925, June to July, it was again faint. The period, if such exists, must be longer than five months.

NEBULAE INVOLVED IN N.G.C. 6822

The only individual objects, other than the variables, which can be definitely identified as members of the system are five diffuse nebulae and the stars with which they are associated. Several other minute nebulous images lie within the limits of the cluster, which may be small, unresolved globular clusters belonging to the system, or, more probably, non-galactic nebulae whose images are superposed on N.G.C. 6822. The diffuse nebulae and the more conspicuous of the supposedly non-galactic nebulae are listed in Table III, under the Roman numerals by which they are designated on Plate XV. The second and third columns give the co-ordinates in millimeters measured from the upper right (north preceding) corner of Plate XV, and the remaining columns the class, diffuse or non-galactic; the general character of the spectrum when known, *E*

denoting emission and *C*, continuous; and short descriptions with angular dimensions as measured on the three-and-a-half-hour exposure with the 100-inch reflector. The linear dimensions are given by the relation $1'' = 1.04$ parsecs or 3.4 light-years. Number V is the object which Wolf and others identified as N.G.C. 6822, and Number X, that catalogued as I.C. 1308.

TABLE III

NEBULAE INVOLVED IN OR SUPERPOSED ON N.G.C. 6822

NEBULA	CO-ORDINATES*		CLASS	SPECTRUM	DESCRIPTION
	Horizontal	Vertical			
	mm	mm			
I.....	8.0	2.5	Dif.	E	32″×48″ several stars involved
II.....	9.5	27.5	Dif.	.....	20″ diam. Incomplete ring around star
III.....	14.0	6.0	Dif.	E	52″ diam. Ring around several stars
IV.....	52.5	95.5	N.-G.	.....	5″ diam. Round, star 2″ n.p. center
V.....	55.0	15.0	Dif.	E	20″×32″ cometic. Stars involved
VI.....	56.0	70.0	N.-G.	C	8″ diam. Round
VII.....	60.5	67.5	N.-G.	C	8″ diam. Round
VIII.....	67.0	15.5	N.-G.	.....	6″ diam. Round
IX.....	81.0	73.0	N.-G.	.....	5″ diam. Round
X.....	83.0	16.0	Dif.	E	22″ diam. Ring around star, fainter extensions to 30″ s.p. star

* Co-ordinates are measured from the upper right (north preceding) corner of Plate XV.

DIFFUSE NEBULAE

The spectral characteristics of several of the nebulae were obtained from slitless spectrograms made at the primary focus of the 60-inch reflector with two prisms and an f 1.9 lens of 3-inch focus. Number V, the brightest of all the nebulae, registered eight definite images with others suspected. The wave-lengths and relative intensities are N_1 (10), N_2 (3), $H\beta$ (5), λ 4471 (1), $H\gamma$ (4), $H\delta$ (2), $H\epsilon$ (1), λ 3727 (1). Number X, the next brightest, registered five images which, with relative intensities, are N_1 (10), N_2 (3), $H\beta$ (5), $H\gamma$ (3), $H\delta$ (1). Numbers I and III registered one faint image each, probably the combination of N_1, N_2 and $H\beta$. The relative intensities of the characteristic lines N_1, N_2 and $H\beta$ in Numbers V and X are those of

the diffuse nebulae in the galactic system and in the Magellanic Clouds.

The mean diameter of the five diffuse nebulae is about 37″, or 41 parsecs, which is comparable with the dimensions of the great loop of nebulosity in the constellation of Orion. The largest nebula, Number III, is a ring about 60 parsecs in diameter. This equals the diameter of N.G.C. 604 in Messier 33 and is of the order of size of the largest known diffuse nebula—N.G.C. 2070 in the large Magellanic Cloud—for which Shapley[1] gives a diameter of 80 parsecs.

A single slit spectrogram of Number V has been obtained under difficult observing conditions, using one prism and a 3-inch camera at the Cassegrain focus of the 100-inch. An exposure on August 16, 1923, registered faint images of N_1 and N_2. Independent measures by Dr. Sanford and the writer gave velocities, uncorrected for solar motion, of +16 and +34 km/sec., respectively. The mean, +25 km/sec., not only gives the order of velocity of nebula Number V but also, judged from the small peculiar motions of diffuse nebulae in general, represents the order of the motion of N.G.C. 6822 as a whole.

The emission character of the diffuse nebulae indicates that the spectral types of the brightest stars involved are very probably Bo or earlier.[2] Such stars are highly luminous, and it is not surprising to find them among the brightest contained in the cluster. Their faint apparent magnitudes combined with the considerable galactic latitude are strong evidence of their association with the extragalactic system. Their presence alone would indicate an enormous distance for N.G.C. 6822.

The photographic magnitudes of the brightest stars involved in the individual nebulae are listed below. The measures required a very uncertain distance correction, and the resulting values may be more accurately described as estimates.

Number	Apparent	Absolute
I.	18.0.....	−3.65
II.	18.0.....	−3.65
III.	16.9.....	−4.75
IV.	16.8.....	−4.85
V.	16.8.....	−4.85

[1] *Harvard Bulletin*, No. 816.

[2] *Mt. Wilson Contr.*, No. 241; *Astrophysical Journal*, 56, 162, 1922.

The absolute magnitudes as computed from the modulus $m-M=21.65$, derived from the Cepheids, are comparable with those of the brightest Bo- and O-type stars in the galactic system. The large range among the latter, and the effects of selection on those in N.G.C. 6822, prevent the use of the data as an accurate criterion, but the comparison confirms the general order of the distance derived from the Cepheids, with evidence of an independent nature. This very materially supports the assumption that the period-luminosity relation functions normally in extra-galactic space.

NON-GALACTIC NEBULAE

The objects in Table III which are classed as "N.G." (non-galactic) are small, round, blurry images whose diameters are functions of the exposure times. The two brightest (Nos. VI and VII) have registered faint continuous streaks on the slitless spectrograms, with no evidence of emission. A dozen or more small non-galactic nebulae are scattered over the reflector field outside the limits of N.G.C. 6822, and it is reasonable to expect that some may be found within the limits, especially as this is the region of best definition. Since, however, normal globular clusters are found in the Magellanic Clouds, there is a possibility that some of the objects in Table III may be the central regions of such clusters actually involved in N.G.C. 6822.

Shapley's[1] value for the average absolute photographic magnitude of globular clusters is about -8.2 ± 0.6. At the distance of N.G.C. 6822, $m-M=21.65$, the apparent photographic magnitude for such a cluster as a whole would be about 13.5 or possibly 14.0 for the denser central portion. The brightest of the objects in N.G.C. 6822, however, is certainly fainter than 16.0, and the mean of the five listed in Table III is considerably more than a magnitude fainter than this. The discrepancy between them and normal globular clusters is almost conclusive.

Shapley reports seven globular clusters in the large Magellanic Cloud,[2] with apparent photographic magnitudes ranging from 7.2 to 12.0, the mean being 9.2. At the distance of N.G.C. 6822, these

[1] *Mt. Wilson Contr.*, No. 152; *Astrophysical Journal*, **48**, 154, 1918.

[2] *Harvard Circular*, No. 271.

values would be increased about four magnitudes. Hence only the faintest, whose real nature is doubtful,[1] would be comparable with even the brightest of the objects in N.G.C. 6822.

The angular diameters of the clusters in the large cloud range from 1′.1 to 2′.6 with a mean value of 1′.8. At the distance of N.G.C. 6822, these quantities would be 11″ to 25″ with a mean of 17″. A glance at Table III shows the diameters of the objects in N.G.C. 6822 ranging from 5″ to 8″ with a mean of about 6″.4. Here again the discrepancy is almost impossibly large.

If a normal globular cluster were associated with N.G.C. 6822, the mean apparent photographic magnitude[2] of the twenty-five brightest stars would be about 20.2. In view of the dense crowding, the unresolved background, and the low observing altitude, it is improbable that such a cluster could be resolved by the 100-inch reflector. The discrepancies in size and brightness between normal globular clusters and the objects photographed with N.G.C. 6822, however, are sufficient to differentiate them, and the latter must be considered as ordinary small non-galactic nebulae whose images are superposed on that of N.G.C. 6822.

DISTRIBUTION OF STELLAR LUMINOSITIES IN N.G.C. 6822

A preliminary attempt has been made to determine the distribution of the brighter absolute photographic magnitudes in N.G.C. 6822 by counting the stars within successive limits of apparent photographic magnitude over the central region, including the core. The area investigated is about 8′ in declination and 4′ in right ascension; more accurately it contains 33.75 square minutes of arc, and, to the limit of the counts, is estimated to cover about one-half of the stars in the entire system. This limit was set at 19.4 and approximate corrections for the systematic error affecting the fainter stars have been derived from minimum magnitudes of the Cepheids, the slope of the period-luminosity curve, and extrapolations of the luminosity function itself. These corrections have little weight, but they are confined to a single interval in which stars were counted and cannot seriously affect the final results.

[1] *Harvard Circular*, No. 271.

[2] *Mt. Wilson Contr.*, No. 152; *Astrophysical Journal*, 48, 154, 1918.

The counts were made on the blink-comparator carrying a scale-plate on one side and a plate of N.G.C. 6822, covered by a reseau, on the other. The scale was calibrated from the series of comparison stars established for the variables, and the field stars were eliminated by means of the new distribution tables for different photographic magnitudes and galactic latitudes recently published by Seares and van Rhijn.[1]

TABLE IV

LUMINOSITY FUNCTION FOR THE BRIGHTER STARS IN N.G.C. 6822

Pg. Mag.	Stars Counted	Normal No.	N.G.C. 6822		Kapteyn*		
		Gal. Lat. −20°	$\phi(m)$	$\log \phi(m)$	$\phi(M)$	$\log \phi(M)$	M (vis.)
19.4−19.1.....	394	13.5	380.5	2.580	379	2.579	−3.75
19.1−18.8.....	230	11.6	218.4	2.339	219	2.340	4.05
18.8−18.5.....	134	9.8	124.2	2.094	124	2.093	4.35
18.5−18.2.....	78	8.3	69.7	1.843	69.2	1.840	4.65
18.2−17.9.....	27	7.0	20.0	1.301	38.2	1.582	4.95
17.9−17.6.....	25	5.9	19.1	1.281	20.7	1.317	5.25
17.6−17.3.....	16	4.9	11.1	1.045	11.1	1.045	5.55
17.3−17.0.....	11	4.1	6.9	0.84	5.9	0.77	5.85
17.0−16.7.....	9	3.3	5.7	.76	3.1	.49	6.15
16.7−16.4.....	9	2.7	6.3	.80	1.6	0.20	6.45
16.4−16.1.....	5	2.2	2.8	.45	0.8	9.89	6.75
16.1−15.8.....	5	1.8	3.2	0.51	.4	9.59	7.05
15.8−15.5.....	0	1.4	− 1.4	.....	0.2	9.29	−7.35
15.5−15.2.....	1	1.1	− 0.1	.....	.....	.....	.....
15.2−14.9.....	0	0.9	− .9	.....	.....	.....	.....
14.9−14.6.....	1	.7	− .3	.....	.....	.....	.....
14.6−14.3.....	0	0.6	− 0.6	.....	.....	.....	.....

* The last three columns give the numbers of stars (visual magnitudes) in a volume of 5.22×10^9 cubic parsecs, in which the distribution is in accordance with Kapteyn's luminosity function for stars in the vicinity of the sun. A comparison with the distribution of stars in N.G.C. 6822 indicates a mean color-index for the latter stars of +1.35 mag.

Results of the counts are given in Table IV. The first three columns represent the number of stars counted in the corresponding intervals of apparent magnitude and the normal number for an equal area in the galactic latitude of the cluster. The fourth and fifth columns give the numbers of stars, $\phi(m)$, with their logarithms, which can be attributed to the cluster. For the brightest stars the results are of low weight because of the small area involved. The remaining columns will be explained later.

The upper limit for the stars in N.G.C. 6822 is about 15.8,

[1] *Proceedings of the National Academy of Sciences*, **11**, 358, 1925.

corresponding to the absolute photographic magnitude -5.85. This appears to be quite normal for the great isolated systems. The brightest star known in the local cluster is -5.5 (β Orionis).[1] The limit for the small Magellanic Cloud[2] has been placed between -4 and -7; for the large cloud,[3] it is brighter than -8; for the globular clusters,[4] fainter than -4.

The smoothness of the luminosity function for N.G.C. 6822 invites a comparison with the corresponding function in the neighborhood of the sun. The latter has been derived by Kapteyn[5] from data based on visual magnitudes. Little is known concerning the distribution of color among the very luminous stars but, as a first approximation, the mean color-index may be assumed to be constant over the short range of magnitudes involved in the present counts. Successive numerical values of the color-index were tried, the two functions being shifted into approximate coincidence by adjusting the volume coefficient[6] for Kapteyn's function.

By this method of trial and error, the best fit was obtained with a mean color-index of $+1.35$ mag., and a volume coefficient of 5.22×10^8 cubic parsecs. The small range of the counts restricts the sensitiveness of the method, but on the assumption that the functions are comparable, the probable error of the color-index, assumed to be constant, is of the order of 0.3 mag. The numbers of stars, $\phi(M)$, and their logarithms, as calculated from Kapteyn's function with the foregoing assumptions, are listed in the sixth and seventh columns of Table IV. The eighth column gives the median absolute visual magnitudes of the stars in the two preceding columns. These magnitudes, corrected by $+1.35$, represent the photographic magnitudes of the stars counted in N.G.C. 6822, using the modulus $m-M=21.65$.

Figure 3 represents the two functions in a graphical form. The

[1] *Mt. Wilson Contr.*, No. 147; *Astrophysical Journal*, **47**, 280, 1918.

[2] *Harvard Circular*, No. 260. [3] *Ibid.*, No. 271.

[4] *Proceedings National Academy of Sciences*, **6**, 293, 1920.

[5] *Mt. Wilson Contr.*, No. 229; *Astrophysical Journal*, **55**, 242, 1922.

[6] The volume coefficient represents the space throughout which stars must be distributed according to Kapteyn's function in order that the number of stars in a given interval of absolute magnitude may equal the number actually counted in the cluster.

full broken line represents the counts in N.G.C. 6822; and the dotted curve, Kapteyn's luminosity function. The agreement, especially in the upper and more reliable portions, is remarkably close; but the comparison involves several assumptions and can scarcely be

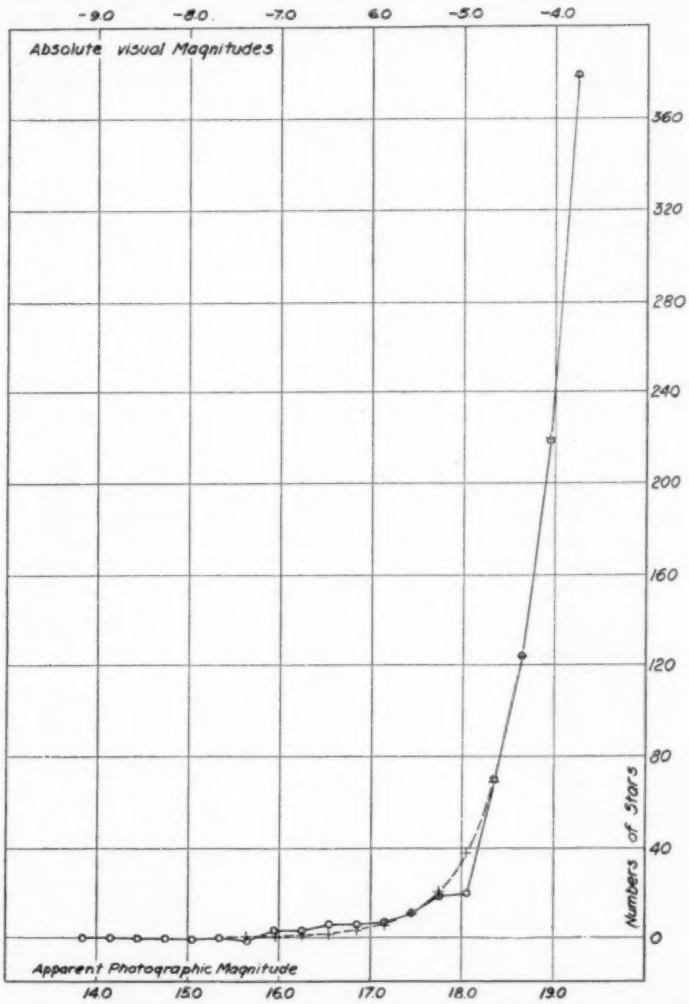


FIG. 3.—Comparison of the luminosity function for N.G.C. 6822 (circles) with Kapteyn's luminosity function (crosses) for stars in the vicinity of the sun. The best fit has been obtained by using a mean color-index of +1.35 mag. for the cluster stars and a volume coefficient of 5.22×10^8 cubic parsecs for Kapteyn's function.

considered as definitive. There is, however, a general similarity between the two functions, and the value of the color-index, judged by the incomplete data available for other systems, seems to be quite reasonable.

The color-index, indeed, appears to confirm the order of the absolute magnitudes assigned to the cluster stars and, consequently, the zero-point of Shapley's general period-luminosity curve for the Cepheids. If the distance assigned the cluster were too great, the color-index as derived from the counts would be too small, and vice versa. An error of 50 per cent in the distance corresponds to an error of about 0.9 mag. in the color-index. There seems to be no reason for supposing that the color-index actually determined from the present data is in error by this amount.

TOTAL MAGNITUDE OF N.G.C. 6822

An approximation to the total photographic magnitude of N.G.C. 6822 has been obtained from short exposures with the 10-inch astrographic camera. The scale of these plates (1 mm = 3′) is so small that the cluster is unresolved and presents an amorphous surface. The brightest portion is the central core, which just registers in an exposure of 30 minutes on a Seed 30 plate. The corresponding surface brightness can be calculated from the value of 18.8±0.3 photographic magnitudes per square second of arc, which Seares[1] has derived for the limiting surface brightness registering in an exposure of 1 minute on a Seed 30 plate at the Newtonian focus of a reflector with a focal ratio of 1 to 5. For an exposure of 30 minutes the limiting surface brightness will be[2]

$$S.B. = 18.8 + 2.0 \log 30 = 21.8$$

This can be reduced to the system for a camera of focal ratio 1:4.5 by applying a correction[3] of +0.3, so that the final value is

$$S.B. = 22.1 \text{ per sq. sec. arc.}$$

[1] *Mt. Wilson Contr.*, No. 191; *Astrophysical Journal*, **52**, 162, 1920.

[2] The coefficient of log exposure time is 2.5 p, where p, the exponent in Schwarzschild's expression for the reciprocity law, is given the usual value of 0.8.

[3] This correction has been applied habitually in deriving surface brightness from limiting exposures with the 10-inch. For an example see *Mt. Wilson Contr.*, No. 250; *Astrophysical Journal*, **56**, 400, 1922.

The area covered by the central core is about $8' \times 3'$, hence the total apparent magnitude is

$$m = 22.1 - 2.5 \log (8 \times 3 \times 3600) = 9.7.$$

It is estimated that the core contributes about one-half the total luminosity of the nebula, hence the total apparent magnitude of the entire system is about 9.0. The corresponding absolute photographic magnitude is -12.7. The absolute visual magnitude is probably of the order of -13.7.

These quantities furnish an indication as to the space density. As a first approximation, the core can be considered as an ellipsoid whose maximum and minimum diameters are those of the projected image and whose third diameter, in the line of sight, is the mean of the other two. Since the linear scale is $1'' = 1.04$ parsecs, the diameters of the hypothetical ellipsoid are about $500 \times 190 \times 340$ parsecs. The volume is 1.7×10^7 cubic parsecs. The absolute photographic magnitude is $9.7 - 21.65 = -11.95$, hence the density is $2.5 \log (1.7 \times 10^7) - 11.95 = 6.1$ absolute photographic magnitudes per cubic parsec. By a similar computation the mean density of the entire system is found to be about 8.8. This is comparable with the density in the neighborhood of the sun,[1] when reasonable assumptions as to color-indices are introduced. The density of the core alone, however, appears to be from twenty to thirty times that near the sun.

A comparison of the total magnitude of the core, derived from the surface brightness, with that derived from the integration of the luminosity function, extrapolated beyond the limits of the star counts previously discussed, furnishes an independent check on the assumption that the latter is comparable with Kapteyn's function for the stars in the vicinity of the sun. The direct comparison of the two luminosity functions was restricted to some three magnitudes at the extreme bright ends of the curves. The excellent agreement over this small region suggests that the two functions are comparable throughout their entire range. In this case, the total luminosity of the portion of the cluster represented by the counts can be computed by integrating Kapteyn's function over a space equal to the

[1] The space density near the sun is about 8.16 absolute visual magnitudes per cubic parsecs. Seares, *Mt. Wilson Contr.*, No. 191, *Astrophysical Journal*, **52**, 162, 1920.

volume coefficient, 5.22×10^8 cubic parsecs, derived in the restricted comparison. The total luminosity per cubic parsec, from Kapteyn's function, is 8.16 in absolute visual magnitudes.[1] The total magnitude of the cluster is, therefore,

$$2.5 \log (5.22 \times 10^8) - 8.15 = -13.64.$$

The core of N.G.C. 6822 covers an area about 25 per cent less than that over which the counts were made, but, as it is much denser than the rest of the cluster, the correction for reducing the volume coefficient to correspond to the core alone can be taken as half this amount. The total magnitude for the core alone is then about -13.5.

This visual luminosity is to be compared with the photographic absolute magnitude, -12.0, derived from the surface brightness and the angular area. The difference of 1.5 mag. represents the combined effect of the mean color-index of the core, errors in the various quantities involved, and any real difference there may be between the two luminosity functions themselves. A large part of the discrepancy can safely be attributed to color-index alone. The comparison of the luminosity functions indicated a mean color-index of $+1.35$ for the brighter stars. This value would probably require some reduction for the fainter stars, but the integrated effect of the factor cannot be accurately stated. The probable error in the surface brightness alone is of the order of ± 0.4 mag., and errors from other sources, although smaller, would add materially to the total.

The discrepancy in the two results, when the color-index is considered, appears to be within the probable errors of the determinations. Nothing remains which can be attributed with certainty to differences in the luminosity functions. As far as one can determine, the stars in N.G.C. 6822 appear to be the same sort of objects, distributed in much the same fashion, as those in the vicinity of the sun.

COMPARISONS OF N.G.C. 6822 WITH THE MAGELLANIC CLOUDS

Comparisons between N.G.C. 6822 and the Magellanic Clouds are of special interest and significance because the distances of all three objects are based primarily on the Cepheid criterion. The

[1] Seares, *Mt. Wilson Contr.*, No. 191; *Astrophysical Journal*, 52, 162, 1920.

appearance of the photographic images is remarkably alike. Shapley[1] has previously noted that "the dominating feature in both [the clouds] is an elongated patch of densely crowded stars similar to N.G.C. 6822." In the present paper, this feature is termed the core. Other points of similarity, which have already been discussed, are

TABLE V

COMPARISON OF N.G.C. 6822 WITH THE MAGELLANIC CLOUDS

	Magellanic Clouds		N.G.C. 6822
	Large	Small	
Angular dimensions:			
Total	7°.2×7°.2*	3°.6×3°.6†	20′×10′
Core	3°.6×1°.2	1°.8×0°.9	8′×3′
Apparent luminosity:			
Total	1.2	2.0‡	9.0
Core	1.9	2.7	9.7
Surface brightness:			
Core	21.0	21.0	22.1
Distance	34,500*	31,600†	214,000
Linear dimensions:			
Total	4300§	2000†	1250×625
Core	2150×715	1100×500†	500×190
Volume:			
Total	4.2×10^{10}	4.2×10^{9}	3.8×10^{8}
Core	9.2×10^{8}	2.3×10^{8}	1.7×10^{7}
Absolute luminosity:			
Total	−16.5§	−15.5	−12.7
Core	−15.8	−14.8	−12.0
Mean density:			
Total	10.0	8.5	8.8
Core	6.6	6.1	6.1

* *H.C.O. Circular*, No. 268. † *H.C.O. Circular*, No. 255. ‡ *H.C.O. Circular*, No. 260.

§ *H.C.O. Bulletin*, No. 816.

Luminosities are expressed in photographic magnitudes; surface brightness, in magnitudes per square second of arc; distances, linear dimensions, and volumes, in parsecs and cubic parsecs; densities, in absolute magnitudes per cubic parsec.

the presence of Cepheid variables and of diffuse nebulae, and the magnitudes and the luminosity function of the brightest stars.

A more detailed comparison of dimensions, densities, and total luminosities is given in Table V, both for the cores and for the entire systems. Many of the data depend upon mere estimates which cannot claim any high accuracy, but even the orders of magnitude of the quantities are useful as a first approximation. Distances, dimensions, and luminosities for the two clouds, when given, have

[1] *Harvard Bulletin*, No. 795.

been taken from Shapley's papers.[1] The angular dimensions of the core of the larger cloud have been measured on the Franklin-Adams chart; the cores of both the clouds have been assumed to contribute one-half the total luminosity of the systems; the true figures of the two clouds are assumed to be globular, since their images are approximately circular; the figures of N.G.C. 6822 and of the cores of all three objects are assumed to be ellipsoids whose maximum and minimum diameters are those of the projected images and whose third diameters are the means of the other two. This last assumption is quite arbitrary, but can serve as a basis for comparison.

CONCLUSION

The present investigation identifies N.G.C. 6822 as an isolated system of stars and nebulae of the same type as the Magellanic Clouds, although somewhat smaller and much more distant. A consistent structure is thus reared on the foundation of the Cepheid criterion, in which the dimensions, luminosities, and densities, both of the system as a whole and of its separate members, are of orders of magnitude which are thoroughly familiar. The distance is the only quantity of a new order.

The principle of the uniformity of nature thus seems to rule undisturbed in this remote region of space. This principle is the fundamental assumption in all extrapolations beyond the limits of known and observable data, and speculations which follow its guide are legitimate until they become self-contradictory. It is therefore a matter of considerable importance that familiar relations are found to be consistent when applied to the first system definitely assigned to the regions outside the galactic system.

Of especial importance is the conclusion that the Cepheid criterion functions normally at this great distance. Cepheid variables have recently been found in the two largest of the spiral nebulae, and the period-luminosity relation places them at distances even more remote than N.G.C. 6822. This criterion seems to offer the means of exploring extra-galactic space; N.G.C. 6822 furnishes a critical test of its value for so ambitious an undertaking and the results are definitely in its favor.

[1] *Harvard Bulletin*, No. 816; *Harvard Circulars*, Nos. 255, 260, 268.

ADDENDUM

A pair of Cramer Iso plates, exposed through a visual color filter at the primary focus of the 100-inch, have recently been obtained of N.G.C. 6822. The exposures were one and three hours, respectively. It has not yet been possible to calibrate accurately the plates, but preliminary comparisons have been made with short, unfiltered exposures obtained at the same time.

Cepheids 1, 2, 3, 4, and 5 were identified and gave indications of moderately large color-indices. The non-Cepheid variables appear to have larger color-indices; 12, 13, and 14 being estimated as red. The diffuse nebulae are clearly blue, and the stars associated with them have color-indices as small as, or smaller than, any others seen on the plates. The colors of the non-galactic nebulae were estimated to fall between those of the Cepheid and the non-Cepheid variables.

The general order of these results probably will not be materially changed when accurate measures are possible. They are thoroughly consistent with the conclusions reached in the previous discussion and furnish additional support to the view that the stars in N.G.C. 6822 are comparable with those in other systems.

MOUNT WILSON OBSERVATORY
September 1925

Minor Contributions and Notes

TWELVE NEW SPECTROSCOPIC BINARIES

ABSTRACT

Data are given for twelve stars of spectral type B, the radial velocities of which have been found to be variable.

The stars listed below have been found during the past year to be spectroscopic binaries. The details for each spectrogram will soon be published in an investigation of the velocities of 368 stars of spectral type B, which have been observed at the Yerkes Observatory for the past 23 years. Most of the measures on which the table is based were made by the writer. About one-half of the observations at the telescope were made by Professor S. B. Barrett.

Star	α 1900	δ 1900	Vis. Mag.	Sp. Type	No. of Plates	Range	Lines
41 Arietis	2^h44^m1	+26°51′	3.7	B8	9	+ 50 to − 14 km/sec.	Few, poor
+48°920 Persei	3 20.9	+48 43	4.9	B5	3	+ 32 − 6	Few, fair
+47°912 Persei	3 48.8	+47 35	5.3	B5	3	+ 42 − 16	Few, fair
11 Orionis	4 58.9	+15 16	4.6	B9	5	+ 38 + 6	Few, good
11 Monocerotis, A	6 24.0	− 6 58	4.7	B3p	3	+ 15 − 2	Few, fair
3 β Can. Min	7 21.7	+ 8 29	3.1	B8	7	+ 53 − 1	Few, poor
16 Argus	8 4.5	−18 57	4.3	B3	10	+ 38 − 48	Many, poor
40 ν^2 Hydrae	10 0.3	−12 35	4.7	B8	6	+ 12 − 53	Few, fair
+20°3481 Herculis	17 22.5	+20 10	5.4	B5	5	+137 −107	Few, poor
66 Ophiuchi	17 55.3	+ 4 23	4.8	B3	5	+101 − 91	Few, poor
+42°3035 Lyrae	18 12.5	+42 8	5.4	B5	5	+ 95 − 96	Few, poor
+55°2639 Cygni	21 48.8	+55 20	5.5	B3	3	+ 77 0	Few, poor

REMARKS

11 Monocerotis is a well-known triple star. The velocity obtained for A is +8.9 km/sec. from the average of three plates. Component B gives +33.8 km/sec. from one plate, and C gives +22.6 km/sec. from two plates. All these stars have strong emission lines of hydrogen.

40 ν^2 Hydrae star was earlier suspected to be a binary by S. B. Barrett and E. C. Arbogast. The lines are double on several plates.

Many spectrograms taken in 1925 of +20°3481 Herculis, 66 Ophiuchi, and +42°3035 Lyrae confirm the duplicity of the lines on some of the earlier plates.

The lines are double on several plates of +55°2639 Cygni.

Otto Struve

Yerkes Observatory

August 12, 1925

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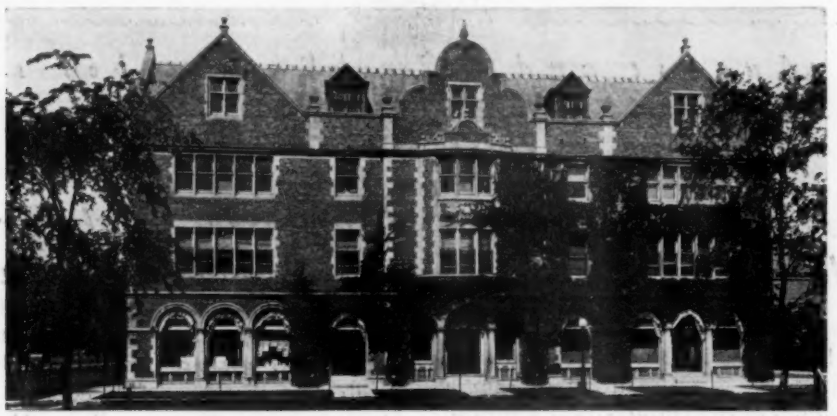

A practical manual for those who write business letters and those who transcribe them is *How to Write Business Letters* by John A. Powell (to be published in September). It is a comprehensive working guide to the everyday problems of the stenographer, and will give letter writers the principles by which effective letters may be written. *HOW TO WRITE BUSINESS LETTERS.* By JOHN A. POWELL. About *200 pages, $1.50, postpaid $1.60.*

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Mr. Boynton is professor of English literature in the University of Chicago, and has for many years been well known for his interpretation of American literature. In this series of sketches, he interprets the drama, the novel, and the poet's art from his background of American life and tradition. He writes of Amy Lowell, Robert Frost, and Bostonia as a New Englander, and of Masters, Sandburg, and Dreiser as a Chicagoan. Very neatly does he take H. L. Mencken's measure. Mr. Huneker the Bohemian is contrasted with Colonel Higginson the Bostonian. An essay on "Biography and the Personal Equation" deals with the treatment of Mark Twain by Albert Bigelow Paine, Van Wyck Brooks, William Dean Howells, and Gamaliel Bradford.

Mr. Boynton sums up Amy Lowell thus: "On the whole, though, in this innocuous revolt, Miss Lowell is a kind of drum major. One cannot see the procession

without seeing her, or admiring the skill with which she swings the baton. But when the parade is past one can easily forget her until the trumpets blare again. She leads the way effectively, and one is glad to have her do it—glad that there are those who enjoy being drum majors. Then one pays farewell to her in the words with which she salutes Ezra Pound in her verses called 'Astigmatism'

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To determine the character of movements and changes of forms of prominences, about four thousand plates were examined. Mr. Pettit analyzes and describes the many interesting types discovered, and discusses the probable nature of eruptive prominences. Eleven plates are included in the book.

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The PRESS IMPRINT

For October and November 1925

The Panchatantra

"One Vishnusharman, shrewdly gleaning
All worldly wisdom's inner meaning,
In these five books the charm compresses
Of all such books the world possesses."

And these five books, *The Panchatantra*, comprising one of the world's best collections of tales, first gathered together over fifteen hundred years ago in the Vale of Kashmir, are now presented for the first time in English, without footnotes and glossaries, simply as stories. One of the great books of the world, akin to *The Arabian Nights* and *Aesop's Fables*, *The Panchatantra* has the glamorous atmosphere of the one and the wisdom of the other. The tales have been translated from the Sanskrit by Arthur W. Ryder; this is, in fact, one of the most important pieces of translation that has been made in the history of Sanskrit studies.

Epigrammatic, philosophical, witty, and wise, these racy tales and shrewd bits of verse take on new life in Arthur Ryder's translation. What Sir Richard Burton did for *The Arabian Nights*, Ryder has done for *The Panchatantra*. In his translation the full oriental flavor has been retained; the translator has carried over complete into English the spirit and form of the original, rendering into English prose and verse the rare beauty of this Eastern masterpiece.

As pure narrative, these tales are charming, and even better than the tales are the verses with their mellow philosophy, wit, and captivating rhythm:

"For men of sense, good poetry
And science will suffice:
The time of dunderheads is spent
In squabbling, sleep, and vice."

The Panchatantra is the artistic representation of a view of life, shrewd, tolerant, and free of all sentimentality; a view that pierces the humbug of every false ideal, revealing with incomparable wit the sources of lasting gratification. These sophisticated and worldly wise tales have but one purpose, which has been brought out for the first time in Arthur Ryder's translation —to teach men how to attain true happiness. The translation opens to the modern occidental reader a magic Eastern realm in which imagination and wit hold full sway.

According to the *legendary* explanation of *The Panchatantra's* origin, this is how it happened:

"In a city called Maiden's Delight, lived a king named Immortal-Power. He had three sons, and they were supreme blockheads. So the king summoned a wise Brahman, one Vishnusharman, and said 'Holy Sir, as a favor to me you must make these princes incomparable masters of the art of practical life.'

"And Vishnusharman answered, 'If I do not in six months' time make the boys acquainted with the art of intelligent living, I will give up my name.' Then he took the princes home, made them learn the five books which he had composed, and in six months the boys answered the prescription. Since that day this work on the art

of intelligent living, *The Panchatantra*, has traveled the world. . . . ."

It is a matter of historical fact, however, that these tales originated in India and were told by Buddhist monks nearly two thousand years ago. In the sixth century they were translated into Pehlevi by Barzuyeh, the physician of Nushirvan, King of Persia. In the middle of the eighth century, they reached Bagdad in the *Kalilah and Dimnah* of Abdullah ibn Almokaffa, later foully murdered by the great Khalif Almansur, just before the reign of Haroun al Raschid of *The Arabian Nights* fame.

From the *Kalilah and Dimnah* the tales were variously translated into Greek, Latin, Hebrew, German, French, and ultimately into the language of practically every European country from Iceland to Italy, from Spain to Constantinople. As separate folk-tales, they became widely known throughout Europe. La Fontaine relied upon them extensively as a source for his fables.

Curiously enough, however, *The Panchatantra* as an entity was never in all these centuries put into adequate English form merely for its intrinsic value. It is true that there were scholarly texts; but in these the wonderful charm of the tales simply as tales was obscured.

And now Arthur Ryder, working from a text written about 1199 A.D., has brought to American readers the complete collection of tales in a translation that preserves the original beauty of these stories and verses which represented the first, most important lessons of worldly wisdom to the people of the lonely little villages of India centuries ago. *THE PANCHATANTRA*. Translated from the Sanskrit by ARTHUR W. RYDER. *$4.00, postpaid $4.15.*

The quintessence of *The Panchatantra*—its wisdom and its charm—is represented in *Gold's Gloom*, a smaller volume of tales from the larger collection which are the high points in its search for what constitutes true happiness.

In essence, the tales offer this philosophy: granted security and freedom from degrading worry, then happiness results from three occupations—from resolute yet circumspect use of the active powers; from intercourse with like-minded friends; and above all from worthy exercise of the intelligence:

"There is no toy
Called easy joy
But man must strain
To body's pain."

This, however, is a philosophic undercurrent which need not even be taken into account for the enjoyment of the tales; they are unexcelled in world-literature simply as tales. Younger readers will find in *Gold's Gloom* the stories that they will like best of all the Panchatantra tales.

These stories bring to the reader the beauty and wisdom, the wit and the color of the complete collection.

Gold's Gloom is bound in exceptionally attractive format, with bright-hued covers, and a specially designed title-page and headbands by Preissig. *GOLD'S GLOOM. Tales from the Panchatantra.* Translated from the Sanskrit by ARTHUR W. RYDER. *$2.00, postpaid $2.10.*

Jonah and the Whale

The modern reader and student finds it difficult to account for such biblical tales as "Jonah and the Whale," "The Fiery Furnace," and "Daniel in the Lions' Den." Professor J. M. Powis Smith points out their true significance in his latest book,

The Prophets and Their Times. He has reconstructed the careers of the Hebrew prophets and the background of political and social events from the days of the newly-born kingdom of Saul, through the dark period of exile, to the re-establishment of the kingdom of Judah in the Holy City. The interpretation of the prophets is to a large extent based upon modern psychology, and uses to advantage the recent discoveries of cuneiform and hieroglyphic records.

The prophets are portrayed as spokesmen of the progressive idealism of their day, organs of a noble discontent with the established order, and heralds of a golden age. From Elijah and Amos to Zachariah and Daniel is a succession of uncompromising upbraiders of their age and champions of a higher ideal—an array of striking personalities which would be hard to equal. *THE PROPHETS AND THEIR TIMES*. By J. M. Powis Smith. *$2.25, postpaid $2.35*.

Mr. Smith is also the author of *The Moral Life of the Hebrews*, a scientific study of the ethics of the ancient Hebrew world. Feeling that the ideals which inspired the Hebrews still point the way for modern civilization, he presents the facts of Hebrew moral life as objectively as possible. The work covers three important periods of Hebraic history: Pre-Prophetic, Prophetic, and Judaistic.

A chapter on "The Morals of the Egyptian Jews" gathers up from the Aramaic papyri discovered a quarter of a century ago the moral aspects of the Jewish-Egyptian life of the fifth century B.C., and presents them with much vividness for the reader of today. Of this book *The Yale Divinity News* says, "An epoch-making work which departs from the pious and transcendental . . . . and interprets Hebrew social life and conduct in the light of history, psychology, and sociology." *THE MORAL LIFE OF THE HEBREWS*. By J. M. Powis Smith. *$2.25, postpaid $2.35*.

Spanish Literature

An Introduction to Spanish Literature by George T. Northup opens for the student and general reader the world of Spanish literature—the realm of picaresque romance, Cervantes, Lope de Vega, and the Cid. This single volume presents a picture of Spanish literature as a whole from its beginning in epic and drama to the present generation of writers. Here is a history of Spanish letters that will not confuse the student with a mass of material, nor fail to give him an adequate selection.

It relates each author to the civilization of his time and brings out clearly what his individual characteristics were and what he contributed. Plots of novels, bits of poetry and story, and critical comment carry the reader along and introduce him to one of the world's great literatures. In the final chapter, after being led through the story of Spanish literature from its earliest days, the reader is introduced to the writers of today—Benavente, Valle-Inclán, Pío Baroja, and others—in a biographical-critical discussion.

Mr. Northup introduces Valle-Inclán thus: "Above all a stylist, he has done for modern Spanish prose what Rubén Darío did for verse. This Galician noble surrounds his early years with so much mystery that one suspects that they were commonplace. He would have us believe that he passed some years in a monastery, served as a soldier of fortune in South America, when, 'on board "La Dalila," I recall it with pride, I assassinated Sir Robert Yones. It was a vengeance worthy of Benevenuto Cellini,

The Origin of the Earth

By

THOMAS C. CHAMBERLIN

The story of a research, the steps and methods of an inquiry, and the exposition of a new theory of the formation of the earth are woven together in this volume. The inquiry started two decades ago in the study of the glacial problem; it led through the enigmas of earlier climatic states, and created grave doubts as to the truth of the conception of a once molten globe enveloped in great, hot atmospheres.

Mr. Chamberlin discusses the disclosures that led to the rejection, one after another, of older views of the origin of the earth, including the nebular hypothesis of Laplace. He describes the futile attempts to emend these theories or to build others on the same foundations, and, finally, traces the steps in the construction of his own, radically new view, based on a dynamic foundation.

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I will tell you how it was, even if you are incapable of comprehending its beauty; but it will be better not to tell you; you might be shocked. . . . .' Today, a long-haired, bespectacled, one-armed bit of picturesqueness, he browbeats and insults admiring café coteries after the fashion of a Dr. Johnson. His arm was lost in an encounter with an antagonist far more formidable than the mythical Sir Robert Yones. He would like to be thought a modern Casanova. His literary forebears, besides Rubén Darío, are Barbey d'Aurevilly, D'Annunzio, and Eca Queiroz; but from these he merely derives hints for the enrichment of a highly original literary personality. . . . ." *AN INTRODUCTION TO SPANISH LITERATURE.* By GEORGE T. NORTHUP. *$3.00, postpaid $3.15.*

* * *

Historical Relativism

A revolution in the world of thought akin to the movement which at the time of the Renaissance freed Western Europe from scholasticism—thus does Florian Znaniecki, Professor of Sociology in the University of Poznan, view the new historical relativism. In *The Laws of Social Psychology* he has applied its principles to the realm of human action—a field from which the traditional methodology has all but retired in defeat. Thus far historical relativism has expressed itself through Nietzsche, Durkheim, Poincaré, Schiller, James, and Dewey, primarily in criticism of traditional dogmas. Mr. Znaniecki's discussion will go far in showing that this revolutionary way of thinking is capable of contributing much to positive scientific and philosophical progress. *THE LAWS OF SOCIAL PSYCHOLOGY.* By FLORIAN ZNANIECKI. *$3.00, postpaid $3.10.*

* * *

A New Series

The initial volume in the "University of Chicago Home Economics Series," *Problems in Home Economics Teaching* by Leona

F. Bowman, has just been published. This series is designed as a contribution to the teaching of home economics in schools, colleges, and universities, and will supply suitable textbooks for secondary schools. It will also give the general reader and especially the educated home-maker material now limited to the classroom.

Problems in Home Economics Teaching is an application of the case or problem method widely and successfully used in other fields, to the teaching of home economics. It presents real cases that will stimulate effective thinking by the student-teacher in methods courses in home economics. *PROBLEMS IN HOME ECONOMICS TEACHING*. By LEONA F. BOWMAN. *$1.50, postpaid $1.60.*

*

A second volume in the series, to be published soon, is *Art in Home Economics: A Bibliography* by Marion E. Clark and others. It is a carefully compiled list of books, articles, and pamphlets on costume design, history of costume, interior decorating, history of furniture, architecture, art principles, and art appreciation. *ART IN HOME ECONOMICS: A BIBLIOGRAPHY*. By MARION E. CLARK and OTHERS. *$1.00, postpaid $1.10.*

* * *

A New Economic Geography

A new type of textbook in economic geography has been developed by Wellington D. Jones and Derwent S. Whittlesey in *An Introduction to Economic Geography*. Its basic organization has the novel feature of making equal use of exercises, textual materials, and illustrations for the approach of each topic. Its arrangement is psychologically sound, and aims primarily to get the student to think. The written and oral exercises require him to use his past experience and present knowledge. Textual readings give new information and associations, and additional exercises fur-

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By

HORATIO HACKETT NEWMAN

In a single volume, a comprehensive account of evolutionary biology —this is what Mr. Newman has achieved in *Evolution, Genetics, and Eugenics*. A textbook for survey courses in the subject, it also offers the reader whose interest has been stimulated by current legal disputes an opportunity to become familiar with the scientific facts and theories involved.

With original material, connective and interpretative sections, Mr. Newman weaves into a unified account a well-balanced selection of excerpts from such writers as Darwin, Wallace, Romanes, DeVries, LeConte, Morgan, Castle, Lull, Guyer, Popenoe, Downing, and Thomson. All the essential reading material for a major or semester course will be found in this single text.

Here, in compact form, is an outline of the development of evolutionary science from Tales and Aristotle to Thomson, Osborn, and Conklin; and a discussion of all the outstanding facts and theories in the field.

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we have recently published **A General Index to Volumes XXVI to L of the Astrophysical Journal.** This **General Index** is of the same size and style as the *Astrophysical Journal* itself and is uniform with the **General Index** to Volumes I to XXV. It should be on every reference table and in every library. It has been compiled by Professor Storrs B. Barrett of Yerkes Observatory, is indexed by author and by subject, and is available now at $2.50 per copy.

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nish training in correlation, investigation, and assimilation. Each step stimulates the faculties of judgment and reflection instead of furnishing mental provender predigested.

Although this text has been available only a few months, it has already been adopted in schools in every part of the country, including Columbia University, University of Minnesota, University of California, University of Oklahoma, and University of Washington.

The book will be in two volumes. Volume I deals with the several elements of the natural environment—climate, natural vegitation, land forms, soils, minerals, surface and ground waters, and size, shape, and location of areas—as they affect economic life. Volume II, to be published in 1926, will be a world-study of the major economic activities as related to natural environment. *AN INTRODUCTION TO ECONOMIC GEOGRAPHY.* Vol. I. By WELLINGTON D. JONES and DERWENT S. WHITTLESEY. *375 pages and 366 illustrations, $5.00, postpaid $5.25.*

* * *

"A Felicitous Essayist"

This is the *Atlantic Monthly's* description of Edgar J. Goodspeed, who this month is the author of a new volume of essays, *Things Seen and Heard.* Here is an excursion of the distinguished translator of the *American New Testament* into the outer world. Mr. Goodspeed writes, with charm and distinction of style, of such things as "The Life of Adventure," "The Week-Ender," "The Spirits of Our Sires," and "Persons and Things." It is a volume that offers much enjoyment to the lover of the personal essay. These papers are the reflection of a charming personality, the product of a mind that plays lightly over the surface of things with delightful humor, and penetrates beneath with wit and gentle irony. *THINGS SEEN AND HEARD.* By EDGAR J. GOODSPEED. *$2.00, postpaid $2.10.*

The PRESS IMPRINT

For October and November 1925

The Panchatantra and the Critics

Arthur W. Ryder's translation of *The Panchatantra* from the Sanskrit presents to American readers for the first time a world-classic akin to the "Arabian Nights," a collection over fifteen hundred years old of some of the best tales that have ever been written. Now, less than two months after publication, the first printing is exhausted, and the book is taking a leading place in the reviewers' columns. Here is how an ancient Eastern masterpiece appears to modern occidental critics:

Harry Hansen, writing in the *Chicago Daily News* under the title "Discretion in Sanskrit" says: "It seems to have been concocted by an ancient people who loved life, who lived with gusto, who profited by their experiences and who knew as much about living 2,000 years before modern plumbing as we do now. Arthur W. Ryder who has translated the whole of *The Panchatantra* for the first time—bits have been done before—evidently believes in the English language as a living tongue, just as Carl Sandburg believes in it. Consequently the book lacks that "literary" tone that would frighten off the younger generation. It seems singularly modern, considering that it was written before Christ."

Under "The Editor Recommends" *The Bookman* offers the following comment: "A series of remarkable and ancient stories which will appeal to any lover of the folkways of mankind. You have but to read these charming fables to learn all about life, and how it should be lived to the greatest advantage and with the least pain. . . . . Mr. Ryder's prose is fine. His translations approved in high quarters are doubtless fine too. . . . . "

Achmed Abdullah, himself of the Orient and a writer of tales, writes in the *New York Evening Post:* "And since legend is always so much more convincing and truthful than stupid historical or literary fact, I hold personally that it was Krishna himself who wrote *The Panchatantra*—Sri Krishna, the flute-playing god who represents so well both the eternal youth and the eternal age of India.

"If mere man had written these tales, mere man would have made himself the hero—and occasionally the villain of these tales. But, being written by a God, they have such delightful heroes and villains as rabbits that get into victorious arguments with jackals; lions that pick their teeth like gentlemen and wear the holy thread of Brahmin and Parohita; flowers that believe in self-expression and make, consequently, a mess of their own and other flowers' lives; monkeys that hold metaphysical discussions with the fleas in their tails. . . . .

"Fairytales. But fairytales for grown-ups. Fairytales suffused with a wisdom that is ironic, slightly bitter, wire-drawn, perfectly civilized.

"Mr. Ryder's translation is most excellent. It shows not only a thorough knowledge of the subject, but understanding, sympathy, style, and a keen sense of humor." *THE PANCHATANTRA.* Translated from the Sanskrit by ARTHUR W. RYDER. *$4.00, postpaid $4.15.*

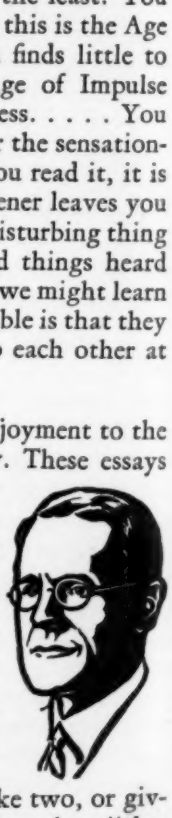
A number of representative tales from the complete work have been published in a

smaller volume, under the title of one of the stories, *Gold's Gloom.* Herschell Brickell, of the *New York Evening Post,* says of these two volumes of tales, "*The Panchatantra* is the sort of source-book no good library—not even an apartment-sized library—can be without; it is a volume as full of sheer story-interest as one could imagine, and at the same time amazingly rich in ripe and seasoned wisdom—old wisdom, which is ever new, as fresh and as useful today as when it was evolved out of life in the ancient East. In fact, its freshness is its most striking quality. *Gold's Gloom,* which contains a selection of tales from the larger book, is an attractive small volume of which only 2,000 copies have been printed. It is intended chiefly as a gift book. Mr. Ryder has performed a real service in giving this fine translation to American readers." *GOLD'S GLOOM, Tales from the Panchatantra.* Translated by ARTHUR W. RYDER. *$2.00, postpaid $2.10.*

Things Seen and Heard

Things Seen and Heard is the new volume of essays by Mr. Edgar J. Goodspeed, famous translator of the New Testament. In his opening essay, giving the theme that runs through much of his writing, Mr. Goodspeed says: "Reflection has satisfied me that much confusion of the modern mind is due to the incongruity of what we hear and what we see. . . . . You look about upon a community of earnest, hard-working people, soberly doing their daily work at business and at home. But you pick up the home edition and read of a very different world of violence and vice. All its men are scoundrels and its women quite different from those you see, to say the least. You have long been assured that this is the Age of Reason; but observation finds little to support the claim. The Age of Impulse would seem as good a guess. . . . . You read much lamentation over the sensationalism of the press, but as you read it, it is its conventionality that oftener leaves you mourning. . . . . The most disturbing thing is not that things seen and things heard contradict each other: that we might learn to allow for. The great trouble is that they seem to bear no relation to each other at all. . . . ."

This volume offers much enjoyment to the lover of the personal essay. These essays are the reflection of a charming personality, the product of a mind that plays lightly over the surface of things with delightful humor, and penetrates beneath with wit and gentle irony.



Whether he is whimsically exposing the shallow and delusive nature of the hoary axiom that one and one make two, or giving his reasons for believing that "theology should always be inculcated to a psychological *obligato,*" or touching lightly upon his college days, Mr. Goodspeed

sees the world as a place not simply of facts and reality, but of something more interesting and intricate, in which "persons are as important as things, and feelings as facts."

"Tame thought, like the caged condor is a sad spectacle"—this is one of Mr. Goodspeed's admonitions which he himself heeds. He is paradoxical without worshiping the paradox, he is epigrammatic without scorning well-timed loquaciousness; he achieves subtlety of idea and expression without loss of appreciation for an occasional bit of homely sentiment. *THINGS SEEN AND HEARD.* By EDGAR J. GOODSPEED. *$2.00, postpaid $2.10.*

* * *

Higher Algebra

For one of the books in the "University of Chicago Science Series," *Algebras and Their Arithmetics*, Leonard E. Dickson was awarded a prize of one thousand dollars by the American Association for the Advancement of Science. Mr. Dickson is regarded by many as one of the world's foremost mathematicians. He has opened up an entirely new science, and has developed a far-reaching theory of the arithmetic of any rational algebra, whose richness and simplicity mark it as the true generalization of the classical theory of algebraic numbers.

The book appeals to wide classes of readers. No effort has been spared in making the text clear and strictly elementary, and it may be read by anyone acquainted with the simpler parts of a first course in the theory of equations. *ALGEBRAS AND THEIR ARITHMETICS.* By LEONARD E. DICKSON. *$2.25, postpaid $2.35.*

* * *

The Nobel Prize

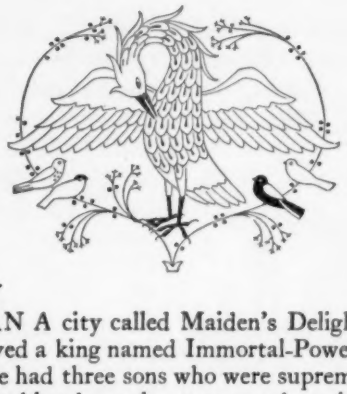
Robert A. Millikan was awarded the Noble Prize in Physics for his researches in the subatomic world which he has described in *The Electron.* Not long ago we brought out a new edition of this widely-known book. It is now completely up to date, including for example even the Compton effect and the recent experiments that bring support to Einstein's quantum theory of radiation. Mr. Millikan has made his style here semi-popular in order to carry his ideas not only to the physicist, but also to the reader of less technical training.

He explains the famous "balanced-droplet" or "oil-drop" method by which he eliminated the uncertainties of preceding work on *e*, and furnished unimpeachable proof of the granular structure of electricity. The book presents, too, a correct and just view of earlier electrical theories and earlier attempts at the determination of *e* from Franklin down to H. A. Wilson. It describes the structure of the atom, and explains how Professor Millikan and other scientists were able to see with the eyes of the mind not only into the atom but also into its nucleus, and to count just how many positive and how many negative electrons are there imprisoned. *THE ELECTRON: Its Isolation and Measurement and the Determination of Some of Its Properties.* By ROBERT A. MILLIKAN. *$2.00, postpaid $2.10.*

* * *

Retail Accounting

Retail accounting and control is a subject that has recently been brought into prominence by the rapid increase in size of the typical retail business. It is, for the student of accounting, a new field with its own special problems. To meet the needs of students in courses in mercantile accounting



IN A city called Maiden's Delight lived a king named Immortal-Power. He had three sons who were supreme blockheads, so he summoned a wise Brahman and said, "Holy Sir, as a favor to me you must make these princes incomparable masters of the art of intelligent living."

So this Brahman, over two thousand years ago, somewhere in the Vale of Kashmir, told the boys a series of tales that are among the great stories of all time.

The Panchatantra

which means "five books" in Sanskrit, has now for the first time been completely translated into English, by Arthur W. Ryder, in a desire to retell these stories as they were first told in India. Here is a collection that rivals the "Arabian Nights" of Haroun al Raschid. *$4*

Gold's Gloom

is a smaller volume than *The Panchatantra* in which a number of representative tales have been gathered together in a particularly attractive binding. It is a handsome collection of some of the most captivating stories in the world. *$2*

At all bookstores or with ten cents extra per volume for postage from

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and of those who have to do with the actual management of a retail business, Mr. Albert C. Hodge has prepared a new text, *Retail Accounting and Control*, the only one to consider records, forms, and procedure in collecting information needed by the manager of a retail establishment.

For every executive of a mercantile business, this book offers concrete analysis of the entire retail structure from the cashier's office to the shipping room. It brings together a wide variety of material illustrative of the problems of mercantile controllership—expense classification and distribution, collection of useful operating data, budgeting, and everyday procedure—that will be valuable not only to the mercantile executive, but also to the prospective certified public accountant. Through its analysis of an unusually interesting phase of control, this book will give the student of business administration, whether or not he intends to make specialized use of the material, a deeper understanding of the control problem in general.

The use of cost statistics and other statistical aids to the control of a retail business is emphasized. Mr. Hodge has selected for intensive treatment the most typical of mercantile establishments, the department store. In analyzing the problems of the comptroller, the author takes up policies, functions, and organization of retail businesses, the types of service they give, and the classification and allocation of their expenses. There are supplementary problems and case materials which give the student an opportunity to see how the principles discussed are applied in actual practice. *RETAIL ACCOUNTING AND CONTROL*. By ALBERT C. HODGE. *$4.00, postpaid $4.15*.

* * *

Christian Living

Right and wrong are not to be determined by abstract formulas but by analysis of the actual situation—this is the theme of Dr. Gerald B. Smith's ethical thought as evidenced in his *Principles of Christian Living*.

In this work he has combined two modern tendencies: the application of the experimental method in the study of human behavior, and the use of the historical method of interpreting Christianity. This historical and psychological approach is a distinctly new note in the study of Christian ethics.

The historical aspects of morality are viewed as the outgrowth of the social experience of the age. Mr. Smith shows the gradual development in moral conceptions, and points out that "the moral education of humanity is never finished." That is, the present and future must be as subject to creative change as the past has been. Ethical ideas must keep pace with changing social conditions.

He turns, then, to moral behavior as an aspect of present-day social life. He analyzes the motives that enter into Christian living today as the individual Christian finds himself a member of various social groups. What should be the Christian's relation to his church? his family? his recreations, life-work, possessions? What sort of amusements contribute to a Christian life? Should the Christian ally himself with an organization for securing a change in the economic order? How should he act in time of war? These and other questions which Dr. Smith takes up are not questions of abstract principle but of practical situation. They represent moral problems which must actually be faced by the individual Christian. *PRINCIPLES OF CHRISTIAN LIVING.* By GERALD B. SMITH. *$2.00, postpaid $2.10.*

*

A new interpretation of the personality of Jesus is promised in the early publication of the Barrows Lectures for 1925, *Jesus and Our Generation*, by Dr. Charles W. Gilkey. Forty thousand people in six great student centers in India heard these lectures, designed in accordance with the purpose of the Barrows Foundation to present "in a friendly, temperate, and conciliatory way" the truths of Christianity to the thought-

To Serve Astronomers

we have recently published **A General Index to Volumes XXVI to L of the Astrophysical Journal.** This **General Index** is of the same size and style as the *Astrophysical Journal* itself and is uniform with the **General Index** to Volumes I to XXV. It should be on every reference table and in every library. It has been compiled by Professor Storrs B. Barrett of Yerkes Observatory, is indexed by author and by subject, and is available now at $2.50 per copy.

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ful people of India. Mr. Gilkey is one of the most popular preachers in America, and his published lectures are assured of a very wide audience, when they appear. *JESUS AND OUR GENERATION.* By CHARLES W. GILKEY. *$2.00, postpaid $2.10.*

* * *

The New Testament Abroad

The much-discussed American Translation of the New Testament by Edgar J. Goodspeed, published by us, is announced for publication in an Indian Edition by the Christian Literature Society of Madras. It is the result of continuous requests from the Orient for a translation of the New Testament in modern English that can be used by the native students, who, in trying to understand the language, find the style of the older versions cumbersome.

In China also the Goodspeed New Testament is soon to be published for native readers of English. There it will be issued by the Commercial Press, Ltd. of Shanghai, from plates recently made in America.

And on November 15, in America, a new edition, printed from new plates, will be issued to replace the present "Popular," or $1.50 edition. This new volume will be called the Tyndale edition, in commemoration of William Tyndale who first translated the New Testament into English just four hundred years ago, in December 1525. *THE NEW TESTAMENT:* An American Translation. By EDGAR J. GOODSPEED. (Tyndale Edition.) $1.50, *postpaid $1.60.*

*

During December, as churches, schools and libraries are celebrating the four hundredth anniversary of Tyndale's epoch-making work, readers of the Tyndale Edition of the Goodspeed New Testament will find particularly interesting the story of how Tyndale and other Bible translators brought this great work to English readers. *THE MAKING OF THE ENGLISH NEW TESTAMENT.* By EDGAR J. GOODSPEED. *$1.50, postpaid $1.60.*

The PRESS IMPRINT

December 1925 — January 1926

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As the East Sees the East

Perhaps the most interesting section of the world today, both for its present curious intermingling of civilizations, and for the developments which must inevitably come about in the future, is the Far East, particularly China and Japan.

Two kinds of observers are viewing the situation in the Far East from widely different angles. There is the Oriental who sees the East with the eyes of the East, and there is the representative of Western civilization who sees the East with the eyes of the West.

The Harris Foundation Lectures for 1925 bring to American readers the views of outstanding thinkers of both types.

The Eastern views are given by Count Michimasa Soyeshima and Dr. P. W. Kuo. Count Soyeshima, who is a member of the House of Peers of Japan, and a leading exponent of Japanese foreign policy, contributes three notable lectures: "The Political, Economic, and Social Aspects of Modern Japan," "Japan's Policy in the Far East," and "Japan's Relations with the United States."

Japan's greatest problems, Count Soyeshima believes, are the domestic ones. Japan, a country given over to a form of medieval feudalism, became, in the lifetime of a man, a great modern power. Analyzing the problems created, Count Soyeshima emphasizes the political aspect—the movement toward democratic government. In his discussion of Japan's economic problems, he draws a startling picture of a Japan that may be confronted with actual starvation within the next thirty years.

He gives the Japanese attitude toward our exclusion laws, and discusses the question of war between Japan and the United States. Japanese relations with soviet Russia, the famous Twenty-one Demands, the Washington Conference, and Japan's past and present policy in China, Siberia, and Korea are all topics in this Japanese view of modern Japan.

*

Dr. Kuo, who is President of Southeastern University, Nanking, China, and one of the best-known educators in that country, does for modern China what Count Soyeshima has done for Japan, describing her political, economic, and social conditions; her position in Eastern Asia; and her relations with the United States.

China is pictured as a land in transition, the meeting place of a new and an old civilization, with the new gradually taking ascendancy over the old. Dr. Kuo discusses the modern tendencies: the development of public opinion, the reorganization of the educational system, labor troubles and the growth of modern industry, the changes in morals and in the position of women, the disintegration of the family unit, and the new culture movement or Chinese Renaissance.

China's attitude toward bolshevism and her relationships with Russia, Japan, and Great Britain with reference to Manchuria, Mongolia, and Tibet are topics in Dr. Kuo's formulation of the position of China in Eastern Asia.

Dr. Kuo treats China's political, economic, and cultural relations with the United States, discussing the Chinese attitude toward our exclusion laws. *ORIENTAL INTERPRETATIONS OF THE FAR EASTERN PROBLEM.* By Count Michimasa Soyeshima and Dr. P. W. Kuo. *$2.00, postpaid $2.10.*

As the West Sees the East

In contradistinction to the oriental interpretations are the views on the Far East given by an English editor from Pekin, an American business man, and an American commercial attaché.

Mr. H. G. W. Woodhead, editor of the *Pekin and Tientsin Times* and an Englishman of twenty years' residence in China, draws a somewhat darker picture of modern China than that presented by Dr. Kuo. He is particularly concerned with acquainting Western readers with the status of the Chinese Republic. To clarify the present situation he tells what has happened in China since the formation of the Republic: the rôles played by Dr. Sun Yat-Sen and Yuan Shih-Kai, the Great War and the Twenty-one Demands, and the recent struggles between rival war lords. It is China with her constant civil warfare, her nominal republic, her millions of illiterates, and her financial bankruptcy, that appears in Mr. Woodhead's treatment.

Personally observed cases of the way in which Chinese law is administered are included in Mr. Woodhead's extensive discussion of the question of extra-territoriality.

American interests in China, commercial and cultural, are taken up, and Mr. Woodhead discusses China's relations with Japan, France, and soviet Russia, going into the question of how far the present disturbances are due to bolshevik instigation.

Henry Kittredge Norton, business man and publicist, writes of the Russians in the Far East. He tells the long story of Russian-Japanese relations with China, and interprets the new treaties. His summary of the situation is that "there in the Far East is going on at this very moment a desperate game of world-politics no less fraught with danger to America and American interests than the game which culminated at Sarajevo in June, 1914."

Julean Arnold, United States consul or commercial attaché in China since 1902, contributes a chapter on China's economic resources. He takes up the question of over-population, compares China with the United States in mechanical development, and points out the difficulties of the Chinese business man. *OCCIDENTAL INTERPRETATIONS OF THE FAR EASTERN PROBLEM.* By H. G. W. Woodhead, C.B.E.; H. K. Norton; and Julean Arnold. *$2.00, postpaid $2.10.*

Colonel Ellsworth

Colonel Elmer E. Ellsworth was one of that patriotic triad of Civil War heroes and martyrs: John Brown, Elmer Ellsworth, and Abraham Lincoln. In view of the vividness of his career, it is surprising that until now, more than sixty years after his death, there has never been an adequate

biography of him. Lest Ellsworth's life-story be permitted to lapse from the memory of the American people, Mr. Charles A. Ingraham introduces us to this remarkable figure in a biography based upon original sources and a personal contact with certain phases of the subject.

Ellsworth's life is depicted from his boyhood days as the village tailor's son until his heroic death at Alexandria at the very

outset of the Civil War. There are interesting sidelights upon the pre-war activities of the Zouave Cadets, including the triumphant tour of the Chicago Zouaves to defend the title of Military Champions of the United States and Canada. *ELMER E. ELLSWORTH AND THE ZOUAVES OF '61.* By CHARLES A. INGRAHAM. *Paper, $2.00, postpaid $2.10.*

* * *

The Panchatantra of the Litterateur

There are a few books in world-literature which more than any others hold a fascination for writers themselves, and one of them is *The Panchatantra*. Since the creation of these wise tales nearly two thousand years ago in the Vale of Kashmir, and throughout the course of their travels over Europe and Western Asia, from England to the Malay Peninsula, they have exercised a strong and direct literary influence. They have many times been paid the compliment of translation whether under the name of "Kalila and Dimna," "Anvar-i-Suhaili," "Directorium Vitae Humanae," "Buch der Beispiele," "Fables of Bidpai," or finally "The Panchatantra" of Arthur W. Ryder. Many writers have given these tales the even greater tribute of pilfering them for material. La Fontaine is perhaps the outstanding example, while in recent times the "Uncle Remus" stories are evidence of the allurement this famous fable book of ancient India holds as a perennial source of literary inspiration.

It is still much too soon after the introduction in America of the complete collection of tales in Arthur W. Ryder's translation of *The Panchatantra* to know what influence they will next exert upon literature and literary men. The following letter to *The Saturday Review of Literature* from the delightful poet and dramatist, Lee Wilson Dodd, indicates that *The Panchatantra* has lost none of its age-old fascination for the creative artist:

"In a world in which so much that is really excellent gets smothered from view and fails to win the attention of the very people most fitted to enjoy it, I think it more and more behooves any one of us who happens upon a treasure to turn himself into as much of a megaphone as he can manage and proclaim his find from the housetops.

"Therefore at the full power of my lungs (which are not so powerful as I could wish) I emit the following yawp:

"Read *The Panchatantra!*

"It is translated joyously by Arthur W. Ryder. It is published by the University of Chicago Press. It costs $4. And it is worth many times that amount. . . . .

"This is rough barking for a fine and ancient and humorous and humane and incredibly wise and lovely book—but O Lord! what a hubbub those Best Sellers are making. So forgive my manners, but remember my advice:

"Read *The Panchatantra!*"

Signed, LEE WILSON DODD

That many people are "reading *The Panchatantra*" is evidenced by the almost immediate sale of the first two impressions; and for those who have not yet read it, a third is in preparation. *THE PANCHATANTRA.* Translated from the Sanskrit by ARTHUR W. RYDER. *$4.00, postpaid $4.15. GOLD'S GLOOM. Tales from the Panchatantra.* Translated by ARTHUR W. RYDER. *$2.00, postpaid $2.10.*

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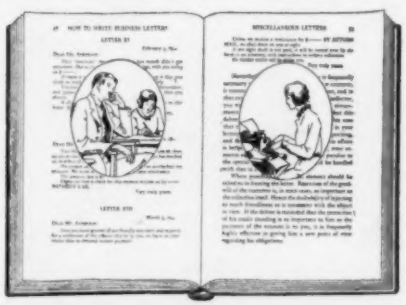

Mr. Powell, more than any other writer on business correspondence, emphasizes the importance of the stenographer in turning

Teaching Science in the Schools

By

ELLIOT R. DOWNING

Scientific study of science teaching has had some valuable results, and these Mr. Downing summarizes in this volume. Here is shown how the growing body of facts about the pedagogy of science, which have been ascertained by the experimental method, may be applied practically to the problems of the science teacher.

This book describes the social and economic background of the science-teaching movement, the history of science teaching in our elementary and secondary schools, present conditions in science teaching, and the aims held.

From this point Mr. Downing takes up the principles of selection and organization of science materials, and actual methods of instruction—by book, lecture-demonstration, and laboratory. There is specific information on notebooks and drawings, field trips, testing results; and suggestions are made for a complete science library in the school.

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out effective business letters, and his is a practical manual both for the man who dictates letters and for the stenographer who transcribes them.

Questions of capitalization, of correct English, of spelling, and of the formal appearance of the letter are constantly coming up in the day's work of a stenographer. How she solves these problems affects greatly the final value of the letter. Mr. Powell's treatment is intended to obviate the danger that the letter will fail to be effective for mechanical reasons.

For the dictator, Mr. Powell goes into such questions as atmosphere and personality, the make-up of a sales letter, and originality of expression; and gives examples of every sort of business letter. Here is a new kind of manual—no hackneyed phrases and devices, nothing about letters with a "punch"—a convincing, individual treatment that will help the letter-dictator write more original and persuasive business letters. *HOW TO WRITE BUSINESS LETTERS.* By JOHN A. POWELL. *$1.50, postpaid $1.60.*

* * *

Science in Politics

Charles E. Merriam is a leader in the movement to apply scientific methods of inquiry to political phenomena. His latest book, *New Aspects of Politics*, is a startling discussion of the effects of contributions from psychology, biology, and sociology upon the present state of political studies. It has been written in the belief that the time has come for adapting the technique of the scientist to politics. Mr. Merriam sums up the point of view upon which the discussion is based when he says "What advantage shall we reap if science conquers the whole world except the world's government?"

Mr. Merriam covers all the various aspects of the new science of politics, foreshadowing developments which are of the future. There are, for example, the possibilities of the application of biological knowledge to political phenomena. Here, Mr. Mer-

riam points out "Control is likely in the future to reach a point where it may be possible to breed whatever type of human being it is desired to have."

Then, there is the intimate association of politics and numbers. Mr. Merriam sums up the possibilities and the limitations of statistical measurement of political phenomena.

Other tendencies in political study are outlined: tendencies in political education, in the organization of adult intelligence, and in the furthering of scientific research in government. Mr. Merriam shows that it would be possible to have a new majority in twenty-one years with an entirely new political education—that the world could be re-created politically in that time through education in the new science of politics.

Here is analysis that will further the development of clearer political thinking and sounder practice. *NEW ASPECTS OF POLITICS.* By CHARLES E. MERRIAM. *$2.50, postpaid $2.60.*

* * *

Jesus Today

Forty thousand people in six leading student centers of India—Bombay, Lucknow, Lahore Calcutta, Rangoon in Burma, and Madras—heard Dr. C. W. Gilkey talk on Jesus as our generation sees him. The lectures, now offered in book form, were made possible by the Barrows Foundation, and were inspired by its purpose of presenting "in a friendly, temperate, and conciliatory way, the truths of Christianity to the scholarly and thoughtful people of India." Dr. Gilkey, one of the most popular preachers in America, is one who speaks with the authentic voice of the younger generation.

And it is to Jesus, himself, as distinguishable from the doctrines, historical movements, and institutions which have taken his name, that Dr. Gilkey turns in these lectures. What he attempts is, in effect, a re-discovery of Jesus. "If you would know what essential Christianity is," says Dr.

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By EDGAR J. GOODSPEED

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Lovers of the personal essay will recognize much potential enjoyment in these titles: "The Life of Adventure," "Do One and One Make Two?" "The House of the Mind," "The New Barbarism," "Democracy Delved Into," "The Self-Made Article," "The Week-Ender." For charm and distinction of style, these essays are worthy of a place with the best in America and abroad.

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Gilkey, "you must not look at our creeds, our churches, our civilization, or ourselves; for neither we nor anything we have made is worthy to represent him. Only he can show you what his religion really is. Look at him!"

In rhythmical prose that retains the eloquence of direct speech, Dr. Gilkey sketches afresh the personality of Jesus, the source of that spiritual power and guidance that the Orient and the Western world alike need, and the sum of what is most vital in the Christian religion. Jesus in his way of life, in his attitude toward various problems, in his life with God; Jesus before the mysteries of life and death; Jesus alive within the forms of historic Christianity; and Jesus living on into the future—Dr. Gilkey touches upon each in succession. He has created that rare thing, a book to which one may turn for a glimpse of the personality of Jesus in its beauty, simplicity, and power. *JESUS AND OUR GENERATION.* By CHARLES W. GILKEY. *$2.00, postpaid $2.10.*

* * *

Prescription Writing

Prescription writing is generally considered by the medical student to be a difficult task. For information on the subject, he is usually referred to one or another of the excellent texts on prescription writing. These texts are comprehensive treatises, and are extremely valuable reference books, but the busy student to whom prescriptions are but a means to an end, finds them too complete and elaborate for the practical necessities of quick and frequent reference.

In *Prescription Notes*, Dr. Arthur L. Tatum has supplied the student and the physician with a simple outline of the elementary principles of prescription writing. A well-made little hand-book in flexible covers, it contains, in addition to the text, blank pages for noting selected prescriptions.

Prescription Notes, with its compact outline of principles, is a useful adjunct to a physician's working library. It is a splendid gift for a doctor, particularly with the recipient's name stamped in gold on the cover. *PRESCRIPTION NOTES.* By ARTHUR L. TATUM. *$1.25, postpaid $1.35.* With your name stamped in gold on the cover *$1.50, postpaid $1.60.*